Midterm Exam R12522615 王邑安 ID:2



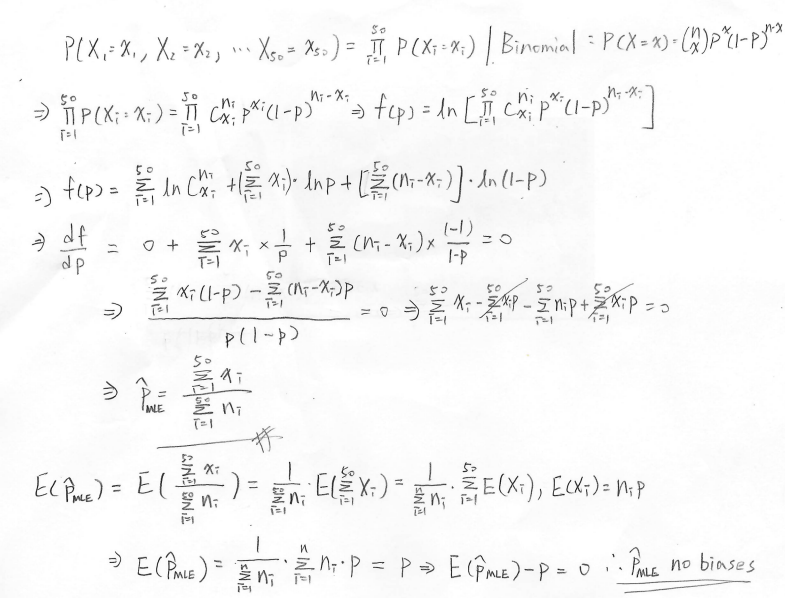
Assumption:

* IDD assumption in each term: Every winning set has identical probability to be chosen independently each term.
* IDD assumption in choosing a number each time: Every number has the identical probability to be chosen independently each time.

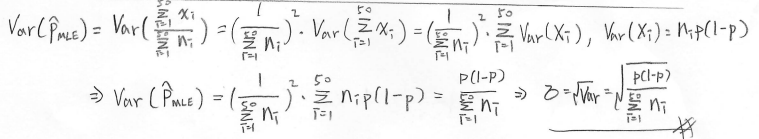
There are totally outcomes in Big Lotto. To win the fourth prize, four of six numbers should be the winning numbers in the first set, and one of the rest numbers should be the special numbers. The probability of fourth prize will be:



In maximum likelihood estimation, will maximize .



By the derivation above,   could be written as the total fourth prize number over the period divided by total trials over the period . Also, could be proven that it has no biases by finding the expected value of . Once was founded, the standard error of it could be estimated by the following formula.



|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 開獎日 | 注數 | 4獎中獎數 | 開獎日 | 注數 | 4獎中獎數 |
| 4月23日 | 1890662 | 78 | 2月16日 | 3766045 | 151 |
| 4月19日 | 1825215 | 83 | 2月15日 | 3946720 | 187 |
| 4月16日 | 1875931 | 80 | 2月14日 | 5440029 | 233 |
| 4月12日 | 1898399 | 89 | 2月13日 | 3807036 | 130 |
| 4月9日 | 2217157 | 139 | 2月12日 | 4978061 | 190 |
| 4月9日 | 2023176 | 76 | 2月11日 | 5421670 | 178 |
| 4月2日 | 2044755 | 77 | 2月10日 | 6228404 | 344 |
| 3月29日 | 1962803 | 93 | 2月9日 | 5782611 | 271 |
| 3月26日 | 1973645 | 87 | 2月8日 | 3673079 | 158 |
| 3月22日 | 2018177 | 90 | 2月7日 | 2942186 | 109 |
| 3月19日 | 2057045 | 111 | 2月6日 | 4308773 | 216 |
| 3月15日 | 2052515 | 100 | 2月2日 | 2179897 | 81 |
| 3月12日 | 2945834 | 100 | 1月30日 | 2135837 | 78 |
| 3月8日 | 2651765 | 100 | 1月26日 | 2048380 | 69 |
| 3月5日 | 2693112 | 143 | 1月23日 | 1859946 | 105 |
| 3月1日 | 2397648 | 89 | 1月19日 | 2058135 | 72 |
| 2月27日 | 2285357 | 107 | 1月16日 | 2091740 | 90 |
| 2月24日 | 2814840 | 150 | 1月12日 | 2094995 | 91 |
| 2月23日 | 3095869 | 153 | 1月9日 | 2088048 | 69 |
| 2月22日 | 2739793 | 127 | 1月5日 | 2025467 | 81 |
| 2月21日 | 2772715 | 119 | 1月2日 | 1883075 | 90 |
| 2月20日 | 3122054 | 146 | 12月29日 | 3156543 | 156 |
| 2月19日 | 2963152 | 133 | 12月26日 | 1956325 | 129 |
| 2月18日 | 3034165 | 120 | 12月22日 | 1848658 | 61 |
| 2月17日 | 3296659 | 148 | 12月19日 | 1924018 | 122 |

The estimated is slightly smaller than . If the sample size is bigger, the difference between and may become smaller.



Assume that the probability of winning a fourth prize follow the binomial distribution. To do the Hypothesis test with , the lower bound could be constructed as the inverse of binomial distribution with and . The upper bound could be constructed as the inverse of binomial distribution with and . The result will reject if the numbers of winning fourth prize exceed the boundaries.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 開獎日 | 注數 | 4獎中獎數 |  | Binom\_0.05(lb) | Binom\_0.95(ub) |  | p value |
| 4月23日 | 1890662 | 78 | 4.50521E-05 | 70 | 101 | accept | 0.474568822 |
| 4月19日 | 1825215 | 83 | 4.50521E-05 | 68 | 97 | accept | 0.874390998 |
| 4月16日 | 1875931 | 80 | 4.50521E-05 | 70 | 100 | accept | 0.673077487 |
| 4月12日 | 1898399 | 89 | 4.50521E-05 | 71 | 101 | accept | 0.656843332 |
| 4月9日 | 2217157 | 139 | 4.50521E-05 | 84 | 117 | reject | 0.000174714 |
| 4月9日 | 2023176 | 76 | 4.50521E-05 | 76 | 107 | accept | 0.118647814 |
| 4月2日 | 2044755 | 77 | 4.50521E-05 | 77 | 108 | accept | 0.12146387 |
| 3月29日 | 1962803 | 93 | 4.50521E-05 | 73 | 104 | accept | 0.581028259 |
| 3月26日 | 1973645 | 87 | 4.50521E-05 | 74 | 105 | accept | 0.894262818 |
| 3月22日 | 2018177 | 90 | 4.50521E-05 | 76 | 107 | accept | 0.978547348 |
| 3月19日 | 2057045 | 111 | 4.50521E-05 | 77 | 109 | reject | 0.055972509 |
| 3月15日 | 2052515 | 100 | 4.50521E-05 | 77 | 109 | accept | 0.400624037 |
| 3月12日 | 2945834 | 100 | 4.50521E-05 | 114 | 152 | reject | 0.003639961 |
| 3月8日 | 2651765 | 100 | 4.50521E-05 | 102 | 138 | reject | 0.076904438 |
| 3月5日 | 2693112 | 143 | 4.50521E-05 | 104 | 140 | reject | 0.048793296 |
| 3月1日 | 2397648 | 89 | 4.50521E-05 | 91 | 125 | reject | 0.068719223 |
| 2月27日 | 2285357 | 107 | 4.50521E-05 | 87 | 120 | accept | 0.64514132 |
| 2月24日 | 2814840 | 150 | 4.50521E-05 | 109 | 146 | reject | 0.039697948 |
| 2月23日 | 3095869 | 153 | 4.50521E-05 | 120 | 159 | accept | 0.23707133 |
| 2月22日 | 2739793 | 127 | 4.50521E-05 | 105 | 142 | accept | 0.704695596 |
| 2月21日 | 2772715 | 119 | 4.50521E-05 | 107 | 144 | accept | 0.636086578 |
| 2月20日 | 3122054 | 146 | 4.50521E-05 | 121 | 160 | accept | 0.614639399 |
| 2月19日 | 2963152 | 133 | 4.50521E-05 | 115 | 153 | accept | 0.988216535 |
| 2月18日 | 3034165 | 120 | 4.50521E-05 | 118 | 156 | accept | 0.161670735 |
| 2月17日 | 3296659 | 148 | 4.50521E-05 | 129 | 169 | accept | 0.99047783 |
| 2月16日 | 3766045 | 151 | 4.50521E-05 | 149 | 191 | accept | 0.159173533 |
| 2月15日 | 3946720 | 187 | 4.50521E-05 | 156 | 200 | accept | 0.463682693 |
| 2月14日 | 5440029 | 233 | 4.50521E-05 | 220 | 271 | accept | 0.462209083 |
| 2月13日 | 3807036 | 130 | 4.50521E-05 | 150 | 193 | reject | 0.001123036 |
| 2月12日 | 4978061 | 190 | 4.50521E-05 | 200 | 249 | reject | 0.021210908 |
| 2月11日 | 5421670 | 178 | 4.50521E-05 | 219 | 270 | reject | 1.03308E-05 |
| 2月10日 | 6228404 | 344 | 4.50521E-05 | 253 | 308 | reject | 0.000222804 |
| 2月9日 | 5782611 | 271 | 4.50521E-05 | 234 | 287 | accept | 0.492755929 |
| 2月8日 | 3673079 | 158 | 4.50521E-05 | 145 | 187 | accept | 0.593736346 |
| 2月7日 | 2942186 | 109 | 4.50521E-05 | 114 | 152 | reject | 0.040364854 |
| 2月6日 | 4308773 | 216 | 4.50521E-05 | 171 | 217 | accept | 0.112135238 |
| 2月2日 | 2179897 | 81 | 4.50521E-05 | 82 | 115 | reject | 0.085379239 |
| 1月30日 | 2135837 | 78 | 4.50521E-05 | 80 | 113 | reject | 0.064385092 |
| 1月26日 | 2048380 | 69 | 4.50521E-05 | 77 | 108 | reject | 0.013783809 |
| 1月23日 | 1859946 | 105 | 4.50521E-05 | 69 | 99 | reject | 0.021672162 |
| 1月19日 | 2058135 | 72 | 4.50521E-05 | 77 | 109 | reject | 0.030277027 |
| 1月16日 | 2091740 | 90 | 4.50521E-05 | 79 | 110 | accept | 0.71118009 |
| 1月12日 | 2094995 | 91 | 4.50521E-05 | 79 | 111 | accept | 0.778620616 |
| 1月9日 | 2088048 | 69 | 4.50521E-05 | 78 | 110 | reject | 0.008297825 |
| 1月5日 | 2025467 | 81 | 4.50521E-05 | 76 | 107 | accept | 0.306737318 |
| 1月2日 | 1883075 | 90 | 4.50521E-05 | 70 | 100 | accept | 0.531204805 |
| 12月29日 | 3156543 | 156 | 4.50521E-05 | 123 | 162 | accept | 0.232900504 |
| 12月26日 | 1956325 | 129 | 4.50521E-05 | 73 | 104 | reject | 3.59799E-05 |
| 12月22日 | 1848658 | 61 | 4.50521E-05 | 69 | 99 | reject | 0.012938894 |
| 12月19日 | 1924018 | 122 | 4.50521E-05 | 72 | 102 | reject | 0.00027628 |



is the number of winning fourth prize in a draw. is the total trials in each draw. The following parameters can be derived as:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | CL | UCL | LCL |
| 4.41845E-05 | 2805962.42 | 3.96812E-06 | 4.42E-05 | 5.61E-05 | 3.23E-05 |

|  |  |  |
| --- | --- | --- |
|  |  |  |
| Binomial.dist | 0.001850711 | 0.000831981 |



In the beginning, there has been the special number in a set. As a result, there are only five numbers we can choose. The outcome space is . Four number should be the winning numbers. The rest of it should not be any winning number. The probability in this situation is:



The probability of typeⅡerror is . is the trial number in each draw. is the shifted probability. and is the upper bound and lower bound constructed in (iii) respectively. And the following parameter is:

the result:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 開獎日 | 注數 | 4獎中獎數 | p\_0 | Binom\_0.05(lb) | Binom\_0.95(ub) | p\_1 | P\_new(lb) | P\_new(ub) | β |
| 4月23日 | 1890662 | 78 | 4.50521E-05 | 70 | 101 | 0.0004 | 6.1E-204 | 1.07E-175 | 1.1E-175 |
| 4月19日 | 1825215 | 83 | 4.50521E-05 | 68 | 97 | 0.0004 | 1.6E-196 | 4.21E-170 | 4.2E-170 |
| 4月16日 | 1875931 | 80 | 4.50521E-05 | 70 | 100 | 0.0004 | 8E-202 | 1.6E-174 | 1.6E-174 |
| 4月12日 | 1898399 | 89 | 4.50521E-05 | 71 | 101 | 0.0004 | 4.6E-204 | 9.34E-177 | 9.3E-177 |
| 4月9日 | 2217157 | 139 | 4.50521E-05 | 84 | 117 | 0.0004 | 5.9E-237 | 6.28E-207 | 6.3E-207 |
| 4月9日 | 2023176 | 76 | 4.50521E-05 | 76 | 107 | 0.0004 | 5E-217 | 8.66E-189 | 8.7E-189 |
| 4月2日 | 2044755 | 77 | 4.50521E-05 | 77 | 108 | 0.0004 | 3.9E-219 | 6.68E-191 | 6.7E-191 |
| 3月29日 | 1962803 | 93 | 4.50521E-05 | 73 | 104 | 0.0004 | 2.5E-211 | 4.77E-183 | 4.8E-183 |
| 3月26日 | 1973645 | 87 | 4.50521E-05 | 74 | 105 | 0.0004 | 6.8E-212 | 1.08E-183 | 1.1E-183 |
| 3月22日 | 2018177 | 90 | 4.50521E-05 | 76 | 107 | 0.0004 | 2.6E-216 | 4.19E-188 | 4.2E-188 |
| 3月19日 | 2057045 | 111 | 4.50521E-05 | 77 | 109 | 0.0004 | 6.7E-221 | 9.64E-192 | 9.6E-192 |
| 3月15日 | 2052515 | 100 | 4.50521E-05 | 77 | 109 | 0.0004 | 3E-220 | 4.02E-191 | 4E-191 |
| 3月12日 | 2945834 | 100 | 4.50521E-05 | 114 | 152 | 0.0004 | 0 | 3.09E-277 | 3.1E-277 |
| 3月8日 | 2651765 | 100 | 4.50521E-05 | 102 | 138 | 0.0004 | 1.6E-281 | 9.36E-249 | 9.4E-249 |
| 3月5日 | 2693112 | 143 | 4.50521E-05 | 104 | 140 | 0.0004 | 1.7E-285 | 9.86E-253 | 9.9E-253 |
| 3月1日 | 2397648 | 89 | 4.50521E-05 | 91 | 125 | 0.0004 | 6.2E-256 | 6.57E-225 | 6.6E-225 |
| 2月27日 | 2285357 | 107 | 4.50521E-05 | 87 | 120 | 0.0004 | 8.9E-244 | 9.66E-214 | 9.7E-214 |
| 2月24日 | 2814840 | 150 | 4.50521E-05 | 109 | 146 | 0.0004 | 5E-298 | 2.39E-264 | 2.4E-264 |
| 2月23日 | 3095869 | 153 | 4.50521E-05 | 120 | 159 | 0.0004 | 0 | 6.86E-292 | 6.9E-292 |
| 2月22日 | 2739793 | 127 | 4.50521E-05 | 105 | 142 | 0.0004 | 3.4E-291 | 1.92E-257 | 1.9E-257 |
| 2月21日 | 2772715 | 119 | 4.50521E-05 | 107 | 144 | 0.0004 | 6E-294 | 2.9E-260 | 2.9E-260 |
| 2月20日 | 3122054 | 146 | 4.50521E-05 | 121 | 160 | 0.0004 | 0 | 1.23E-294 | 1.2E-294 |
| 2月19日 | 2963152 | 133 | 4.50521E-05 | 115 | 153 | 0.0004 | 0 | 9.17E-279 | 9.2E-279 |
| 2月18日 | 3034165 | 120 | 4.50521E-05 | 118 | 156 | 0.0004 | 0 | 5.75E-286 | 5.7E-286 |
| 2月17日 | 3296659 | 148 | 4.50521E-05 | 129 | 169 | 0.0004 | 0 | 0 | 0 |
| 2月16日 | 3766045 | 151 | 4.50521E-05 | 149 | 191 | 0.0004 | 0 | 0 | 0 |
| 2月15日 | 3946720 | 187 | 4.50521E-05 | 156 | 200 | 0.0004 | 0 | 0 | 0 |
| 2月14日 | 5440029 | 233 | 4.50521E-05 | 220 | 271 | 0.0004 | 0 | 0 | 0 |
| 2月13日 | 3807036 | 130 | 4.50521E-05 | 150 | 193 | 0.0004 | 0 | 0 | 0 |
| 2月12日 | 4978061 | 190 | 4.50521E-05 | 200 | 249 | 0.0004 | 0 | 0 | 0 |
| 2月11日 | 5421670 | 178 | 4.50521E-05 | 219 | 270 | 0.0004 | 0 | 0 | 0 |
| 2月10日 | 6228404 | 344 | 4.50521E-05 | 253 | 308 | 0.0004 | 0 | 0 | 0 |
| 2月9日 | 5782611 | 271 | 4.50521E-05 | 234 | 287 | 0.0004 | 0 | 0 | 0 |
| 2月8日 | 3673079 | 158 | 4.50521E-05 | 145 | 187 | 0.0004 | 0 | 0 | 0 |
| 2月7日 | 2942186 | 109 | 4.50521E-05 | 114 | 152 | 0.0004 | 0 | 9.81E-277 | 9.8E-277 |
| 2月6日 | 4308773 | 216 | 4.50521E-05 | 171 | 217 | 0.0004 | 0 | 0 | 0 |
| 2月2日 | 2179897 | 81 | 4.50521E-05 | 82 | 115 | 0.0004 | 1.4E-233 | 1.64E-203 | 1.6E-203 |
| 1月30日 | 2135837 | 78 | 4.50521E-05 | 80 | 113 | 0.0004 | 3.1E-229 | 3.66E-199 | 3.7E-199 |
| 1月26日 | 2048380 | 69 | 4.50521E-05 | 77 | 108 | 0.0004 | 1.2E-219 | 2.13E-191 | 2.1E-191 |
| 1月23日 | 1859946 | 105 | 4.50521E-05 | 69 | 99 | 0.0004 | 1.6E-200 | 3.56E-173 | 3.6E-173 |
| 1月19日 | 2058135 | 72 | 4.50521E-05 | 77 | 109 | 0.0004 | 4.7E-221 | 6.84E-192 | 6.8E-192 |
| 1月16日 | 2091740 | 90 | 4.50521E-05 | 79 | 110 | 0.0004 | 6.7E-224 | 1.19E-195 | 1.2E-195 |
| 1月12日 | 2094995 | 91 | 4.50521E-05 | 79 | 111 | 0.0004 | 2.3E-224 | 2.96E-195 | 3E-195 |
| 1月9日 | 2088048 | 69 | 4.50521E-05 | 78 | 110 | 0.0004 | 2.3E-224 | 3.81E-195 | 3.8E-195 |
| 1月5日 | 2025467 | 81 | 4.50521E-05 | 76 | 107 | 0.0004 | 2.4E-217 | 4.21E-189 | 4.2E-189 |
| 1月2日 | 1883075 | 90 | 4.50521E-05 | 70 | 100 | 0.0004 | 7.5E-203 | 1.69E-175 | 1.7E-175 |
| 12月29日 | 3156543 | 156 | 4.50521E-05 | 123 | 162 | 0.0004 | 0 | 1.13E-297 | 1.1E-297 |
| 12月26日 | 1956325 | 129 | 4.50521E-05 | 73 | 104 | 0.0004 | 2.1E-210 | 3.67E-182 | 3.7E-182 |
| 12月22日 | 1848658 | 61 | 4.50521E-05 | 69 | 99 | 0.0004 | 6.7E-199 | 1.24E-171 | 1.2E-171 |
| 12月19日 | 1924018 | 122 | 4.50521E-05 | 72 | 102 | 0.0004 | 9.5E-207 | 2.02E-179 | 2E-179 |



The probability of typeⅡerror is . is the trial number in each draw. is the shifted probability. And the following parameter is:

the result:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | ) | β |  |
| 0.000368 | 5.7171E-254 | 0 | 5.7E-254 | 1 |



A chi-squared proportion test with is constructed to estimate the accident death per month in Taoyuan. The number is 84 because there are 84 months from Jan.2011 to Dec.2017. The maximum death in a month is 17 and the minimum death is 4. As a result, 16 kinds of death number (~3,4,…,17,18~) had been tested. The Poisson parameter is 10.

|  |  |  |  |
| --- | --- | --- | --- |
| n | max death number | min death number | k |
| 84 | 17 | 4 | 16 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| death number |  | frequency | (poisson λ=10) |  |  |
| ~3 | 0 | 0 | 0.010336051 | 0.868228 | 0.868228257 |
| 4 | 1 | 0.011905 | 0.018916637 | 1.588998 | 0.218325136 |
| 5 | 3 | 0.035714 | 0.037833275 | 3.177995 | 0.009969257 |
| 6 | 9 | 0.107143 | 0.063055458 | 5.296658 | 2.58931901 |
| 7 | 9 | 0.107143 | 0.090079226 | 7.566655 | 0.271517337 |
| 8 | 6 | 0.071429 | 0.112599032 | 9.458319 | 1.26449199 |
| 9 | 10 | 0.119048 | 0.125110036 | 10.50924 | 0.024676224 |
| 10 | 13 | 0.154762 | 0.125110036 | 10.50924 | 0.590325148 |
| 11 | 8 | 0.095238 | 0.113736396 | 9.553857 | 0.252722262 |
| 12 | 9 | 0.107143 | 0.09478033 | 7.961548 | 0.13544893 |
| 13 | 5 | 0.059524 | 0.072907946 | 6.124267 | 0.206388336 |
| 14 | 5 | 0.059524 | 0.052077104 | 4.374477 | 0.089445967 |
| 15 | 4 | 0.047619 | 0.03471807 | 2.916318 | 0.402688275 |
| 16 | 1 | 0.011905 | 0.021698794 | 1.822699 | 0.371335698 |
| 17 | 1 | 0.011905 | 0.012763996 | 1.072176 | 0.004858652 |
| 18~ | 0 | 0 | 0.014277614 | 1.19932 | 1.199319542 |

The result shows that the accident death per month in Taoyuan over the period followed a Poisson distribution with .



Suppose the accident death per month in Taoyuan followed a Poisson distribution. is the probability of x in Poisson distribution. The upper bound and lower bound in the Hypothesis test with can be constructed by the following:

Since Poisson is a discrete distribution, the upper bound and lower bound in the Hypothesis test with are not really accurate. I use the p-value to do the Hypothesis test. If the p-value is smaller than α=0.05, the death number in that month reject H0.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| month | death in Taoyuan | p-value | H\_0 | month | death in Taoyuan | p-value | H\_0 |
| 1-Jan | 7 | 0.4404413 | accept | 1-Mar | 18 | 0.01437301 | reject |
| 1-Feb | 7 | 0.4404413 | accept | 1-Apr | 16 | 0.05408322 | accept |
| 1-Mar | 5 | 0.1341719 | accept | 1-May | 18 | 0.01437301 | reject |
| 1-Apr | 4 | 0.0585054 | accept | 1-Jun | 7 | 0.44044129 | accept |
| 1-May | 9 | 0.9158594 | accept | 1-Jul | 15 | 0.09748081 | accept |
| 1-Jun | 7 | 0.4404413 | accept | 1-Aug | 14 | 0.16691695 | accept |
| 1-Jul | 10 | 0.8339205 | accept | 1-Sep | 15 | 0.09748081 | accept |
| 1-Aug | 6 | 0.2602828 | accept | 1-Oct | 13 | 0.27107115 | accept |
| 1-Sep | 14 | 0.1669169 | accept | 1-Nov | 18 | 0.01437301 | reject |
| 1-Oct | 6 | 0.2602828 | accept | 1-Dec | 22 | 0.00059147 | reject |
| 1-Nov | 13 | 0.2710712 | accept | 1-Jan | 18 | 0.01437301 | reject |
| 1-Dec | 5 | 0.1341719 | accept | 1-Feb | 10 | 0.8339205 | accept |
| 1-Jan | 21 | 0.0013993 | reject | 1-Mar | 24 | 9.3899E-05 | reject |
| 1-Feb | 13 | 0.2710712 | accept | 1-Apr | 13 | 0.27107115 | accept |
| 1-Mar | 13 | 0.2710712 | accept | 1-May | 12 | 0.41688705 | accept |
| 1-Apr | 11 | 0.6064477 | accept | 1-Jun | 13 | 0.27107115 | accept |
| 1-May | 22 | 0.0005915 | reject | 1-Jul | 19 | 0.00690868 | reject |
| 1-Jun | 17 | 0.0285552 | reject | 1-Aug | 9 | 0.91585943 | accept |
| 1-Jul | 9 | 0.9158594 | accept | 1-Sep | 15 | 0.09748081 | accept |
| 1-Aug | 16 | 0.0540832 | accept | 1-Oct | 24 | 9.3899E-05 | reject |
| 1-Sep | 22 | 0.0005915 | reject | 1-Nov | 14 | 0.16691695 | accept |
| 1-Oct | 16 | 0.0540832 | accept | 1-Dec | 12 | 0.41688705 | accept |
| 1-Nov | 11 | 0.6064477 | accept | 1-Jan | 18 | 0.01437301 | reject |
| 1-Dec | 17 | 0.0285552 | reject | 1-Feb | 15 | 0.09748081 | accept |
| 1-Jan | 21 | 0.0013993 | reject | 1-Mar | 23 | 0.00024024 | reject |
| 1-Feb | 24 | 9.39E-05 | reject |  |  |  |  |



The upper bound in the Hypothesis test is 17, and the lower bound is 4. The type Пerror probability is in Poisson distribution with .



The parameter is the accident death in Taoyuan cities per month. The parameter because there are totally 84 months from Jan.2011 to Dec.2017. The is estimated by the following equation:

The standard error is the square root of . Finally, CL, UCL, and LCL can be constructed:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **σ** | **CL** | **UCL** | **LCL** |
| 10 | 3.139609 | 9.857143 | 19.27597 | 0.438317 |

Use the central line and the two control limits to monitor the number of accident deaths from Jan.2018 to March 2022.

The probability of typeⅠerror is:

where is the probability of x in Poisson with . And ].

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | α |  |
| 0.002954 | 5.24E-05 | 0.003007 | 332.5889 |



The average thicknesses and the standard error of right side and left side could be calculated by the 85-wafers sample in bottom zone.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| average | | | | |
| up | middle | down | left | right |
| 349.7176 | 349.6471 | 356.1882 | 348.7412 | 357.0941 |
| sample variance | | | | |
| 23.75266 | 15.15966 | 142.1308 | 138.1941 | 32.56246 |
| standard error | | | | |
| 4.87367 | 3.893541 | 11.92186 | 11.7556 | 5.706353 |

The t-distribution could be used in this Hypothesis test. The DOF in t-distribution is because there are 85 numbers in a sample. The is 0.02. Consequently, the result will reject if or .



The proportion between two sample variances might be used in the Hypothesis test. The smaller on should be the denominator. F-test could be used to test whether the result reject or not. The result will accept if .



The maximum and minimum are considered as extreme values in those 85 numbers. Therefore, only the values between the second biggest number and the second smallest number are counted precisely in the histogram.

The data in the Q-Q plot has been rearranged from the smallest to the largest. The {x , y} in Q-Q plot is {, th smallest sample observation}. The Gamma distribution has been assumed in this data. The parameter of Gamma can be estimated by following:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| average | sample variance | standard error | estimated α | estimated β |
| 348.7412 | 138.1941 | 11.7556 | 880.0694 | 0.396266 |

To do the proportion test, the appearance numbers of 27 kinds thickness value counted in the histogram must be compared with the probability of those value in Gamma distribution. The probability of those is:

Where is , and follow Gamma distribution with and .

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| thickness | number | (gamma.dist) |  | thickness | number | (gamma.dist) |  |
| x<=333 | 1 | 0.088757 | 7.544311 | 346<x<=347 | 5 | 0.033525 | 2.849623 |
| 333<x<=334 | 1 | 0.014932 | 1.269207 | 347<x<=348 | 2 | 0.033854 | 2.877624 |
| 334<x<=335 | 1 | 0.01664 | 1.414405 | 348<x<=349 | 10 | 0.033939 | 2.884836 |
| 335<x<=336 | 0 | 0.018399 | 1.563887 | 349<x<=350 | 6 | 0.033779 | 2.871222 |
| 336<x<=337 | 0 | 0.020185 | 1.715725 | 350<x<=351 | 12 | 0.033379 | 2.837191 |
| 337<x<=338 | 0 | 0.021974 | 1.867759 | 351<x<=352 | 9 | 0.032748 | 2.783587 |
| 338<x<=339 | 0 | 0.023737 | 2.017643 | 352<x<=353 | 5 | 0.031902 | 2.711644 |
| 339<x<=340 | 1 | 0.025446 | 2.162909 | 353<x<=354 | 4 | 0.030858 | 2.622949 |
| 340<x<=341 | 1 | 0.027071 | 2.301029 | 354<x<=355 | 2 | 0.02964 | 2.519379 |
| 331<x<=342 | 0 | 0.028582 | 2.429491 | 355<x<=356 | 4 | 0.028271 | 2.403041 |
| 342<x<=343 | 2 | 0.029951 | 2.545875 | 356<x<=357 | 4 | 0.026779 | 2.276195 |
| 343<x<=344 | 5 | 0.031152 | 2.647928 | 357<x<=358 | 1 | 0.02519 | 2.141192 |
| 344<x<=345 | 4 | 0.032161 | 2.733644 | 358<x | 1 | 0.214193 | 18.20639 |
| 345<x<=346 | 4 | 0.032957 | 2.80132 | k=27 |  |  |  |

The result shows that the thickness wasn’t follow the Gamma distribution.



Sample mean and standard error are estimated from first 45 wafers. The control lines are constructed as following:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | sample variance | standard error σ | CL | UCL | LCL |
| 354.3911 | 26.34174 | 5.13242 | 354.3911 | 369.7884 | 338.9939 |

3 thickness values exceeded the boundary of  chart.

Sample mean of range and standard error are estimated from first 45 wafers. The control lines are constructed as following:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | sample variance | standard error σ | CL | UCL | LCL |
| 9.822222 | 6.740404 | 2.596229 | 9.822222 | 17.61091 | 2.033536 |

One R value exceeded the boundary of chart.

Remove the extreme wafers in the previous  chart and chart, and construct a chart and chart again with the rest of 41 wafers.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | sample variance | standard error σ | CL | UCL | LCL |
| 354.9707 | 6.561122 | 2.561469 | 354.9707 | 362.6551 | 347.2863 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | sample variance | standard error σ | CL | UCL | LCL |
| 9.902439 | 5.290244 | 2.300053 | 9.902439 | 16.8026 | 3.00228 |

The result shows that there isn’t any value exceed the boundary in chart and chart. The CL, UCL, and LCL in both previous charts can be used to construct the Shewhart control chart to monitor the last 40 wafers.

It is obvious that there is a wafer out of range in the both charts. Most of the wafers’ are lower than the control line, and some wafers’ are lower than the control bound in chart.



In the sequential likelihood ratio test with , is 350, standard error could be estimated by , is , and could be 1 to 40. must be constructed by the following equation. ( will be reset to zero if exceed the boundaries of sequential likelihood ratio test. )

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| average | sample variance | standard error σ |  |  | Δ | α | β | accept (if lower) | accept (if higher) |
| 354.3911 | 26.34174 | 5.13242 | 350 | 341.7881 | -8.21187 | 0.003 | 0.2 | -17.9186 | 5.153057 |

In the “graphical” Tabular CUSUM chart, . and can be constructed by the following equation:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| average | sample variance | standard error σ |  | k | K | h | H |
| 354.3911 | 26.34174 | 5.13242 | 350 | 0.8 | 4.105936 | 6 | 30.79452 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| average | sample variance | standard error σ |  | k | K | h | H |
| 354.3911 | 26.34174 | 5.13242 | 350 | 0.5 | 2.56621 | 5 | 25.6621 |

Comparison:

Sequential likelihood ratio test shows that most of the tend to close to . It accepts and rejects . Between wafers 45 and 58, there are some C^+ become positive in second Tabular CUSUM chart. Similar pattern can be found in the first Tabular CUSUM chart from wafer 45 to 55. However, the majority of both and values are zero, suggesting that the process remains in control. There is an out -of-control signal in Tabular CUSUM chart with . The new process mean is : . Since the , and begin to decrease dramatically, it seems that only wafer 48 out of control, not the total process.



In the optimal Tabular CUSUM chart, to compare the with different shifted means, and should be fixed. Only is changed. and can be found by following:

Comparison:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| shift | δ | k\* | h' |  |
| 0.5σ | 0.5 | 0.8 | 4.475606 | 60.87891 |
| 1.0σ | 1 | 0.8 | 4.475606 | 11.96443 |
| 1.5σ | 1.5 | 0.8 | 4.475606 | 5.375253 |
| 2.0σ | 2 | 0.8 | 4.475606 | 3.382457 |

In the optimal Tabular CUSUM chart, is estimated by the first 45 wafers, , , . and can be constructed by the following equation:

There is an out-of-control signal in optimal Tabular CUSUM chart. The new process mean is :



In the optimal EWMA chart, to compare the with different shifted means, and should be fixed.

and could be found by the following equation:

could be estimated by different equations, depend on the value of :

Comparison:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| shift | δ | λ\* | L\* | g | w | φ(w) | Φ(w) |  |
| 0.5σ | 0.5 | 0.232431 | 3.12491 | 2.130589 | 2.055716 | 0.048223 | 0.980095 | 42.53612 |
| 1.0σ | 1 | 0.232431 | 3.12491 | 1.065295 | 0.753675 | 0.300306 | 0.774478 | 14.72196 |
| 1.5σ | 1.5 | 0.232431 | 3.12491 | 0.710196 | -0.58667 | 0.33587 | 0.278711 | 5.806391 |
| 2.0σ | 2 | 0.232431 | 3.12491 | 0.532647 | -1.94685 | 0.059961 | 0.025776 | 3.95144 |

When , the in EWMA is smaller than CUSUM by about 18.343. This indicates that EWMA could be more sensitive to a slight shift in the mean compared to CUSUM. However, when , the in CUSUM drop sharply to 11.964 which is smaller than the in EWMA. Both in CUSUM and EWMA are small when mean shift become bigger.

In the optimal EWMA chart, could be calculated by the equation:

I suppose it is zero state, therefor, .

The control limits are:

There is also an out-of-control signal in EWMA chart. Both chats indicate that wafer 48 dose not meet the requirement. Nevertheless, the majority of wafer’s values tend to stay in the control limits in both charts, suggesting that the process remains in control.



To estimate all 425 thickness readings in the bottom zone, the is the average of all 425 values, and the is the standard error of total 425 values. USL and LSL are 335 and 360 respectively.

could be calculated by the following equation:

could be calculated by the following equation:

is the target which is , and is the sample average . could found by following:

could be calculated by the following equation:

could be calculated by the following equation:

Out-of-spec% could be estimated by the following equations:

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | sample variance | standard error |  | USL | LSL |  |  |
| 352.2776 | 350 | 82.62084 | 9.089601 | 9.37062 | 360 | 335 | 0.849581 | -1.90081 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  | out-of-spec% |
| 0.458399 | 0.283194 | 0.444652 | 0.355722 | 22.64% |

To improve the process to achieve overall , sample average should be reduced, and should be smaller. The following table present various combination of and making the achieve 2.0.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | 352.2776 | 351.0832 | 349.8888 | 348.6944 | 347.5 |
| σ | 1.286904 | 1.485969 | 1.685159 | 1.883921 | 2.083314 |
|  | 2.000241 | 2.000213 | 2.000044 | 2.000365 | 2.000019 |

To improve the process to achieve overall , should become smaller. The equation indicates that should be reduced. On the other hand, should be as close to target as possible.



To estimate the “within-wafer” reading, the is the average of five thickness values in each wafer, and the is the standard error of five thickness values in each wafer. USL and LSL are 335 and 360 respectively.

could be calculated by the following equation:

could be calculated by the following equation:

is the target which is , and is the sample average . could found by following:

could be calculated by the following equation:

could be calculated by the following equation:

Out-of-spec% could be estimated by the following equations:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| wafer number | X\_bar | σ\_X | σ^~ | Z\_U | Z\_L |
| 1 | 354.6 | 3.209361 | 5.608921 | 1.682578 | -6.10713 |
| 2 | 353.8 | 3.114482 | 4.913247 | 1.9907 | -6.03632 |
| 3 | 353.6 | 3.130495 | 4.770744 | 2.044405 | -5.94155 |
| 4 | 357.6 | 3.847077 | 8.518216 | 0.62385 | -5.87459 |
| 5 | 353.8 | 2.774887 | 4.705316 | 2.234325 | -6.77505 |
| 6 | 352.4 | 2.19089 | 3.249615 | 3.46891 | -7.94198 |
| 7 | 355.6 | 5.029911 | 7.527284 | 0.874767 | -4.0955 |
| 8 | 353.2 | 3.563706 | 4.789572 | 1.908126 | -5.10704 |
| 9 | 355 | 3.24037 | 5.958188 | 1.543033 | -6.17213 |
| 10 | 354.4 | 3.435113 | 5.582114 | 1.630223 | -5.64756 |
| 11 | 354 | 3 | 5 | 2 | -6.33333 |
| 12 | 353.2 | 4.207137 | 5.28583 | 1.616301 | -4.32598 |
| 13 | 354.2 | 5.357238 | 6.807349 | 1.082647 | -3.58394 |
| 14 | 353.8 | 4.868265 | 6.175759 | 1.273554 | -3.86175 |
| 15 | 354.6 | 4.929503 | 6.742403 | 1.095445 | -3.97606 |
| 16 | 353.2 | 4.494441 | 5.517246 | 1.51298 | -4.04945 |
| 17 | 370.4 | 3.507136 | 20.69928 | -2.96538 | -10.0937 |
| 18 | 357.2 | 4.147288 | 8.309031 | 0.67514 | -5.3529 |
| 19 | 359.4 | 6.0663 | 11.18749 | 0.098907 | -4.02222 |
| 20 | 356.6 | 5.412947 | 8.535807 | 0.628124 | -3.99043 |
| 21 | 357.8 | 5.674504 | 9.645724 | 0.387699 | -4.01797 |
| 22 | 356.8 | 3.03315 | 7.445804 | 1.055009 | -7.18725 |
| 23 | 360.6 | 4.97996 | 11.71153 | -0.12048 | -5.1406 |
| 24 | 360.2 | 4.969909 | 11.34637 | -0.04024 | -5.07051 |
| 25 | 358.8 | 5.357238 | 10.30243 | 0.223996 | -4.44259 |
| 26 | 338 | 4.949747 | 12.98075 | 4.444671 | -0.60609 |
| 27 | 354.6 | 5.899152 | 7.480642 | 0.915386 | -3.32251 |
| 28 | 349.2 | 0.83666 | 1.157584 | 12.90847 | -16.9722 |
| 29 | 356.6 | 5.549775 | 8.623224 | 0.612637 | -3.89205 |
| 30 | 357 | 5.09902 | 8.660254 | 0.588348 | -4.31455 |
| 31 | 356.8 | 5.761944 | 8.912912 | 0.555368 | -3.78345 |
| 32 | 355.4 | 6.580274 | 8.512344 | 0.699059 | -3.10018 |
| 33 | 354.8 | 6.180615 | 7.825599 | 0.84134 | -3.20356 |
| 34 | 352.6 | 6.426508 | 6.932532 | 1.151481 | -2.73866 |
| 35 | 351 | 5.244044 | 5.338539 | 1.716233 | -3.05108 |
| 36 | 351.2 | 1.923538 | 2.267157 | 4.574902 | -8.42198 |
| 37 | 348 | 4.795832 | 5.196152 | 2.502173 | -2.71069 |
| 38 | 357.8 | 4.816638 | 9.167333 | 0.45675 | -4.73359 |
| 39 | 354.6 | 5.458938 | 7.138627 | 0.989203 | -3.59044 |
| 40 | 352.4 | 3.286335 | 4.069398 | 2.312606 | -5.29465 |
| 41 | 354.6 | 4.97996 | 6.779381 | 1.084346 | -3.93577 |
| 42 | 336.2 | 4.816638 | 14.61643 | 4.941206 | -0.24914 |
| 43 | 352.6 | 5.412947 | 6.004998 | 1.367093 | -3.25146 |
| 44 | 356 | 5.522681 | 8.154753 | 0.724286 | -3.8025 |
| 45 | 353.4 | 4.27785 | 5.46443 | 1.542831 | -4.30123 |
| 46 | 355.2 | 4.868265 | 7.123202 | 0.985978 | -4.14932 |
| 47 | 355.6 | 4.560702 | 7.222188 | 0.964764 | -4.51685 |
| 48 | 317.6 | 55.55448 | 64.31221 | 0.763215 | 0.313206 |
| 49 | 359 | 4.582576 | 10.0995 | 0.218218 | -5.23723 |
| 50 | 354 | 4.582576 | 6.082763 | 1.309307 | -4.14614 |
| 51 | 354.8 | 4.764452 | 6.763135 | 1.091416 | -4.15578 |
| 52 | 357 | 4.690416 | 8.42615 | 0.639602 | -4.69042 |
| 53 | 351.8 | 5.263079 | 5.562374 | 1.558023 | -3.19205 |
| 54 | 353.4 | 5.128353 | 6.153048 | 1.286963 | -3.5879 |
| 55 | 352.2 | 4.868265 | 5.342284 | 1.602214 | -3.53309 |
| 56 | 348.6 | 1.516575 | 2.063977 | 7.516937 | -8.96757 |
| 57 | 349 | 1.581139 | 1.870829 | 6.957011 | -8.85438 |
| 58 | 349.4 | 5.128353 | 5.163332 | 2.066941 | -2.80792 |
| 59 | 349.4 | 6.14817 | 6.177378 | 1.72409 | -2.34216 |
| 60 | 352.6 | 5.029911 | 5.662155 | 1.471199 | -3.49907 |
| 61 | 353.2 | 4.32435 | 5.379591 | 1.572491 | -4.20873 |
| 62 | 353 | 3.674235 | 4.743416 | 1.905159 | -4.89898 |
| 63 | 350 | 5.244044 | 5.244044 | 1.906925 | -2.86039 |
| 64 | 351.8 | 5.263079 | 5.562374 | 1.558023 | -3.19205 |
| 65 | 352.4 | 5.176872 | 5.706137 | 1.468068 | -3.3611 |
| 66 | 344.2 | 2.683282 | 6.390618 | 5.888312 | -3.42864 |
| 67 | 350.4 | 4.722288 | 4.739198 | 2.032913 | -3.26113 |
| 68 | 349.2 | 4.494441 | 4.565085 | 2.402968 | -3.15946 |
| 69 | 348.2 | 4.38178 | 4.737088 | 2.692969 | -3.01247 |
| 70 | 348.6 | 4.09878 | 4.331282 | 2.781315 | -3.31806 |
| 71 | 350.8 | 4.868265 | 4.933559 | 1.88979 | -3.24551 |
| 72 | 343.8 | 3.49285 | 7.116179 | 4.638047 | -2.51943 |
| 73 | 349.4 | 5.176872 | 5.211526 | 2.047569 | -2.7816 |
| 74 | 348.2 | 4.91935 | 5.23832 | 2.398691 | -2.68328 |
| 75 | 346.8 | 5.263079 | 6.159545 | 2.508038 | -2.24203 |
| 76 | 351 | 5.477226 | 5.567764 | 1.643168 | -2.92119 |
| 77 | 346.8 | 4.38178 | 5.425864 | 3.012474 | -2.69297 |
| 78 | 349.6 | 5.458938 | 5.473573 | 1.905133 | -2.67451 |
| 79 | 350.6 | 5.412947 | 5.4461 | 1.736577 | -2.88198 |
| 80 | 351 | 5.612486 | 5.700877 | 1.603567 | -2.85079 |
| 81 | 349.4 | 5.366563 | 5.4 | 1.975193 | -2.68328 |
| 82 | 351.4 | 5.59464 | 5.767148 | 1.537186 | -2.93138 |
| 83 | 345.2 | 1.788854 | 5.122499 | 8.273452 | -5.70197 |
| 84 | 353.4 | 4.722288 | 5.818935 | 1.397628 | -3.89642 |
| 85 | 348 | 4.582576 | 5 | 2.618615 | -2.83683 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| wafer number | C\_p | C\_pk | C\_pm | C\*\_pm | out-of-spec% |
| 1 | 1.298285 | 0.560859 | 0.742864 | 0.594291 | 0.046228429 |
| 2 | 1.337836 | 0.663567 | 0.848047 | 0.678438 | 0.023256939 |
| 3 | 1.330993 | 0.681468 | 0.873379 | 0.698703 | 0.020456778 |
| 4 | 1.083073 | 0.20795 | 0.489148 | 0.391318 | 0.266362955 |
| 5 | 1.501562 | 0.744775 | 0.885523 | 0.708419 | 0.012730851 |
| 6 | 1.901814 | 1.156303 | 1.282203 | 1.025762 | 0.000261288 |
| 7 | 0.828378 | 0.291589 | 0.553542 | 0.442833 | 0.190871397 |
| 8 | 1.169195 | 0.636042 | 0.869946 | 0.695956 | 0.028187629 |
| 9 | 1.285861 | 0.514344 | 0.699318 | 0.559454 | 0.061411324 |
| 10 | 1.212964 | 0.543408 | 0.746432 | 0.597145 | 0.051527193 |
| 11 | 1.388889 | 0.666667 | 0.833333 | 0.666667 | 0.022750132 |
| 12 | 0.990381 | 0.538767 | 0.788271 | 0.630617 | 0.053022203 |
| 13 | 0.777764 | 0.360882 | 0.612084 | 0.489667 | 0.139651703 |
| 14 | 0.855883 | 0.424518 | 0.674681 | 0.539745 | 0.101466972 |
| 15 | 0.845251 | 0.365148 | 0.617979 | 0.494384 | 0.136695872 |
| 16 | 0.927071 | 0.504327 | 0.755208 | 0.604166 | 0.065168021 |
| 17 | 1.188054 | -0.98846 | 0.201295 | 0.161036 | 0.998488468 |
| 18 | 1.004673 | 0.225047 | 0.501462 | 0.40117 | 0.24979347 |
| 19 | 0.686855 | 0.032969 | 0.37244 | 0.297952 | 0.460634853 |
| 20 | 0.769759 | 0.209375 | 0.48814 | 0.390512 | 0.264994466 |
| 21 | 0.734279 | 0.129233 | 0.43197 | 0.345576 | 0.349148727 |
| 22 | 1.373709 | 0.35167 | 0.559599 | 0.447679 | 0.14571066 |
| 23 | 0.836687 | -0.04016 | 0.355775 | 0.28462 | 0.547949824 |
| 24 | 0.838379 | -0.01341 | 0.367225 | 0.29378 | 0.516050174 |
| 25 | 0.777764 | 0.074665 | 0.404435 | 0.323548 | 0.411384645 |
| 26 | 0.841794 | 0.202031 | 0.320988 | 0.25679 | 0.27223139 |
| 27 | 0.706316 | 0.305129 | 0.556993 | 0.445595 | 0.180440639 |
| 28 | 4.980119 | 4.302823 | 3.599452 | 2.879561 | 6.58965E-65 |
| 29 | 0.750781 | 0.204212 | 0.483191 | 0.386553 | 0.270107737 |
| 30 | 0.817151 | 0.196116 | 0.481125 | 0.3849 | 0.278157227 |
| 31 | 0.723136 | 0.185123 | 0.467487 | 0.373989 | 0.289398787 |
| 32 | 0.633206 | 0.23302 | 0.489485 | 0.391588 | 0.243224585 |
| 33 | 0.674151 | 0.280447 | 0.532441 | 0.425952 | 0.200757364 |
| 34 | 0.648356 | 0.383827 | 0.601031 | 0.480825 | 0.127851784 |
| 35 | 0.794552 | 0.572078 | 0.780488 | 0.624391 | 0.044199832 |
| 36 | 2.166147 | 1.524967 | 1.837838 | 1.47027 | 2.38221E-06 |
| 37 | 0.86881 | 0.834058 | 0.801875 | 0.6415 | 0.009528875 |
| 38 | 0.865057 | 0.15225 | 0.454512 | 0.36361 | 0.323926427 |
| 39 | 0.763274 | 0.329734 | 0.583679 | 0.466943 | 0.161446861 |
| 40 | 1.267876 | 0.770869 | 1.023902 | 0.819122 | 0.010372206 |
| 41 | 0.836687 | 0.361449 | 0.614609 | 0.491687 | 0.139147152 |
| 42 | 0.865057 | 0.083045 | 0.285067 | 0.228054 | 0.40162801 |
| 43 | 0.769759 | 0.455698 | 0.693866 | 0.555093 | 0.086372216 |
| 44 | 0.754465 | 0.241429 | 0.510949 | 0.40876 | 0.234516716 |
| 45 | 0.97401 | 0.514277 | 0.762507 | 0.610006 | 0.061444371 |
| 46 | 0.855883 | 0.328659 | 0.584943 | 0.467954 | 0.162088717 |
| 47 | 0.913602 | 0.321588 | 0.576926 | 0.461541 | 0.167334699 |
| 48 | 0.075001 | -0.1044 | 0.064788 | 0.05183 | 0.845605599 |
| 49 | 0.909241 | 0.072739 | 0.412561 | 0.330049 | 0.413629755 |
| 50 | 0.909241 | 0.436436 | 0.684996 | 0.547997 | 0.095232038 |
| 51 | 0.874532 | 0.363805 | 0.616085 | 0.492868 | 0.137561092 |
| 52 | 0.888336 | 0.213201 | 0.494492 | 0.395594 | 0.261217006 |
| 53 | 0.791679 | 0.519341 | 0.749081 | 0.599265 | 0.060320194 |
| 54 | 0.812477 | 0.428988 | 0.677171 | 0.541737 | 0.09922027 |
| 55 | 0.855883 | 0.534071 | 0.779941 | 0.623953 | 0.054759554 |
| 56 | 2.747419 | 2.505646 | 2.018757 | 1.615005 | 2.80888E-14 |
| 57 | 2.635231 | 2.319004 | 2.227177 | 1.781742 | 1.73783E-12 |
| 58 | 0.812477 | 0.68898 | 0.806972 | 0.645578 | 0.021863013 |
| 59 | 0.677708 | 0.574697 | 0.674504 | 0.539603 | 0.051932029 |
| 60 | 0.828378 | 0.4904 | 0.73588 | 0.588704 | 0.07085208 |
| 61 | 0.963536 | 0.524164 | 0.774532 | 0.619626 | 0.057931228 |
| 62 | 1.134023 | 0.635053 | 0.87841 | 0.702728 | 0.028380205 |
| 63 | 0.794552 | 0.635642 | 0.794552 | 0.635642 | 0.030380755 |
| 64 | 0.791679 | 0.519341 | 0.749081 | 0.599265 | 0.060320194 |
| 65 | 0.804862 | 0.489356 | 0.730208 | 0.584166 | 0.071431025 |
| 66 | 1.552825 | 1.142879 | 0.651997 | 0.521598 | 0.000303311 |
| 67 | 0.882341 | 0.677638 | 0.879192 | 0.703354 | 0.021585498 |
| 68 | 0.927071 | 0.800989 | 0.912725 | 0.73018 | 0.008921608 |
| 69 | 0.950907 | 0.897656 | 0.879584 | 0.703667 | 0.004836578 |
| 70 | 1.016563 | 0.927105 | 0.961994 | 0.769595 | 0.003160182 |
| 71 | 0.855883 | 0.62993 | 0.844556 | 0.675645 | 0.029979197 |
| 72 | 1.192913 | 0.839811 | 0.58552 | 0.468416 | 0.005878964 |
| 73 | 0.804862 | 0.682523 | 0.79951 | 0.639608 | 0.023005704 |
| 74 | 0.846995 | 0.799564 | 0.79542 | 0.636336 | 0.011872073 |
| 75 | 0.791679 | 0.747345 | 0.676457 | 0.541165 | 0.018549786 |
| 76 | 0.760726 | 0.547723 | 0.748355 | 0.598684 | 0.051917626 |
| 77 | 0.950907 | 0.897656 | 0.767927 | 0.614341 | 0.004836578 |
| 78 | 0.763274 | 0.635044 | 0.761233 | 0.608987 | 0.032123312 |
| 79 | 0.769759 | 0.578859 | 0.765074 | 0.612059 | 0.043206863 |
| 80 | 0.742392 | 0.534522 | 0.730882 | 0.584705 | 0.056585277 |
| 81 | 0.776412 | 0.658398 | 0.771605 | 0.617284 | 0.027768282 |
| 82 | 0.74476 | 0.512395 | 0.722483 | 0.577986 | 0.063811258 |
| 83 | 2.329237 | 1.900658 | 0.813405 | 0.650724 | 5.92142E-09 |
| 84 | 0.882341 | 0.465876 | 0.716053 | 0.572843 | 0.081161259 |
| 85 | 0.909241 | 0.872872 | 0.833333 | 0.666667 | 0.006692555 |

To improve the process to achieve “within-wafer” :

* The in each wafer should be close to .
* The in each wafer should be as small as possible.

The following graph compare the in each wafer with .

The graph indicates that most of the wafers’ is bigger than . The process may be adjusted to reduce the SiO2 thickness of wafers and lessen their standard errors, aiming to achieve “within-wafer”.

To improve the process to achieve “within-wafer”, the  in each wafer should be smaller. Since the equation , there are some solutions:

* The in each wafer should be close to .
* The in each wafer should be as small as possible.