Midterm Exam R12522615 王邑安 ID:2



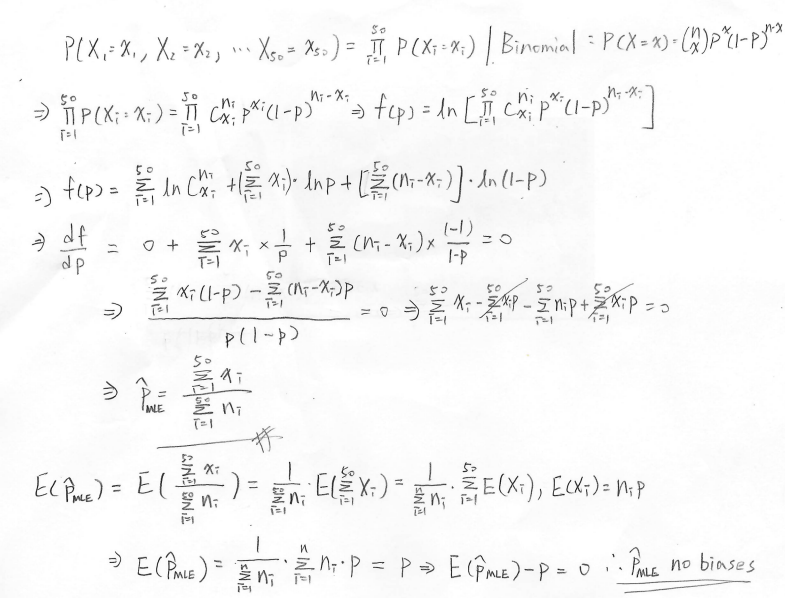
Assumption:

* IDD assumption in each term: Every winning set has identical probability to be chosen independently each term.
* IDD assumption in choosing a number each time: Every number has the identical probability to be chosen independently each time.

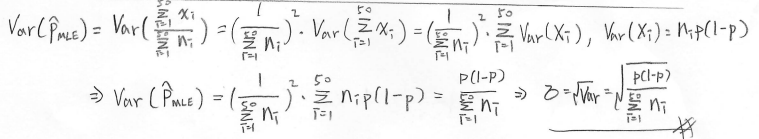
There are totally outcomes in Big Lotto. To win the forth prize, four of six numbers should be the winning numbers in the first set, and one of the rest numbers should be the special numbers. The probability of fourth prize will be:



In maximum likelihood estimation, will maximize .



By the derivation above,   could be written as the total forth prize number over the period divided by total trials over the period . Also, could be proven that it has no biases by finding the expected value of . Once was founded, the standard error of it could be estimated by the following formula.



|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 開獎日 | 注數 | 4獎中獎數 | 開獎日 | 注數 | 4獎中獎數 |
| 4月23日 | 1890662 | 78 | 2月16日 | 3766045 | 151 |
| 4月19日 | 1825215 | 83 | 2月15日 | 3946720 | 187 |
| 4月16日 | 1875931 | 80 | 2月14日 | 5440029 | 233 |
| 4月12日 | 1898399 | 89 | 2月13日 | 3807036 | 130 |
| 4月9日 | 2217157 | 139 | 2月12日 | 4978061 | 190 |
| 4月9日 | 2023176 | 76 | 2月11日 | 5421670 | 178 |
| 4月2日 | 2044755 | 77 | 2月10日 | 6228404 | 344 |
| 3月29日 | 1962803 | 93 | 2月9日 | 5782611 | 271 |
| 3月26日 | 1973645 | 87 | 2月8日 | 3673079 | 158 |
| 3月22日 | 2018177 | 90 | 2月7日 | 2942186 | 109 |
| 3月19日 | 2057045 | 111 | 2月6日 | 4308773 | 216 |
| 3月15日 | 2052515 | 100 | 2月2日 | 2179897 | 81 |
| 3月12日 | 2945834 | 100 | 1月30日 | 2135837 | 78 |
| 3月8日 | 2651765 | 100 | 1月26日 | 2048380 | 69 |
| 3月5日 | 2693112 | 143 | 1月23日 | 1859946 | 105 |
| 3月1日 | 2397648 | 89 | 1月19日 | 2058135 | 72 |
| 2月27日 | 2285357 | 107 | 1月16日 | 2091740 | 90 |
| 2月24日 | 2814840 | 150 | 1月12日 | 2094995 | 91 |
| 2月23日 | 3095869 | 153 | 1月9日 | 2088048 | 69 |
| 2月22日 | 2739793 | 127 | 1月5日 | 2025467 | 81 |
| 2月21日 | 2772715 | 119 | 1月2日 | 1883075 | 90 |
| 2月20日 | 3122054 | 146 | 12月29日 | 3156543 | 156 |
| 2月19日 | 2963152 | 133 | 12月26日 | 1956325 | 129 |
| 2月18日 | 3034165 | 120 | 12月22日 | 1848658 | 61 |
| 2月17日 | 3296659 | 148 | 12月19日 | 1924018 | 122 |

The estimated is slightly smaller than . If the sample size is bigger, the difference between and may become smaller.



Assume that the probability of winning a fourth prize follow the binomial distribution. To do the Hypothesis test with , the lower bound could be constructed as the inverse of binomial distribution with and . The upper bound could be constructed as the inverse of binomial distribution with and . the result will reject if the numbers of winning fourth prize exceed the boundary.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| 開獎日 | 注數 | 4獎中獎數 |  | Binom\_0.05(lb) | Binom\_0.95(ub) |  | p value |
| 4月23日 | 1890662 | 78 | 4.50521E-05 | 70 | 101 | accept | 0.474568822 |
| 4月19日 | 1825215 | 83 | 4.50521E-05 | 68 | 97 | accept | 0.874390998 |
| 4月16日 | 1875931 | 80 | 4.50521E-05 | 70 | 100 | accept | 0.673077487 |
| 4月12日 | 1898399 | 89 | 4.50521E-05 | 71 | 101 | accept | 0.656843332 |
| 4月9日 | 2217157 | 139 | 4.50521E-05 | 84 | 117 | reject | 0.000174714 |
| 4月9日 | 2023176 | 76 | 4.50521E-05 | 76 | 107 | accept | 0.118647814 |
| 4月2日 | 2044755 | 77 | 4.50521E-05 | 77 | 108 | accept | 0.12146387 |
| 3月29日 | 1962803 | 93 | 4.50521E-05 | 73 | 104 | accept | 0.581028259 |
| 3月26日 | 1973645 | 87 | 4.50521E-05 | 74 | 105 | accept | 0.894262818 |
| 3月22日 | 2018177 | 90 | 4.50521E-05 | 76 | 107 | accept | 0.978547348 |
| 3月19日 | 2057045 | 111 | 4.50521E-05 | 77 | 109 | reject | 0.055972509 |
| 3月15日 | 2052515 | 100 | 4.50521E-05 | 77 | 109 | accept | 0.400624037 |
| 3月12日 | 2945834 | 100 | 4.50521E-05 | 114 | 152 | reject | 0.003639961 |
| 3月8日 | 2651765 | 100 | 4.50521E-05 | 102 | 138 | reject | 0.076904438 |
| 3月5日 | 2693112 | 143 | 4.50521E-05 | 104 | 140 | reject | 0.048793296 |
| 3月1日 | 2397648 | 89 | 4.50521E-05 | 91 | 125 | reject | 0.068719223 |
| 2月27日 | 2285357 | 107 | 4.50521E-05 | 87 | 120 | accept | 0.64514132 |
| 2月24日 | 2814840 | 150 | 4.50521E-05 | 109 | 146 | reject | 0.039697948 |
| 2月23日 | 3095869 | 153 | 4.50521E-05 | 120 | 159 | accept | 0.23707133 |
| 2月22日 | 2739793 | 127 | 4.50521E-05 | 105 | 142 | accept | 0.704695596 |
| 2月21日 | 2772715 | 119 | 4.50521E-05 | 107 | 144 | accept | 0.636086578 |
| 2月20日 | 3122054 | 146 | 4.50521E-05 | 121 | 160 | accept | 0.614639399 |
| 2月19日 | 2963152 | 133 | 4.50521E-05 | 115 | 153 | accept | 0.988216535 |
| 2月18日 | 3034165 | 120 | 4.50521E-05 | 118 | 156 | accept | 0.161670735 |
| 2月17日 | 3296659 | 148 | 4.50521E-05 | 129 | 169 | accept | 0.99047783 |
| 2月16日 | 3766045 | 151 | 4.50521E-05 | 149 | 191 | accept | 0.159173533 |
| 2月15日 | 3946720 | 187 | 4.50521E-05 | 156 | 200 | accept | 0.463682693 |
| 2月14日 | 5440029 | 233 | 4.50521E-05 | 220 | 271 | accept | 0.462209083 |
| 2月13日 | 3807036 | 130 | 4.50521E-05 | 150 | 193 | reject | 0.001123036 |
| 2月12日 | 4978061 | 190 | 4.50521E-05 | 200 | 249 | reject | 0.021210908 |
| 2月11日 | 5421670 | 178 | 4.50521E-05 | 219 | 270 | reject | 1.03308E-05 |
| 2月10日 | 6228404 | 344 | 4.50521E-05 | 253 | 308 | reject | 0.000222804 |
| 2月9日 | 5782611 | 271 | 4.50521E-05 | 234 | 287 | accept | 0.492755929 |
| 2月8日 | 3673079 | 158 | 4.50521E-05 | 145 | 187 | accept | 0.593736346 |
| 2月7日 | 2942186 | 109 | 4.50521E-05 | 114 | 152 | reject | 0.040364854 |
| 2月6日 | 4308773 | 216 | 4.50521E-05 | 171 | 217 | accept | 0.112135238 |
| 2月2日 | 2179897 | 81 | 4.50521E-05 | 82 | 115 | reject | 0.085379239 |
| 1月30日 | 2135837 | 78 | 4.50521E-05 | 80 | 113 | reject | 0.064385092 |
| 1月26日 | 2048380 | 69 | 4.50521E-05 | 77 | 108 | reject | 0.013783809 |
| 1月23日 | 1859946 | 105 | 4.50521E-05 | 69 | 99 | reject | 0.021672162 |
| 1月19日 | 2058135 | 72 | 4.50521E-05 | 77 | 109 | reject | 0.030277027 |
| 1月16日 | 2091740 | 90 | 4.50521E-05 | 79 | 110 | accept | 0.71118009 |
| 1月12日 | 2094995 | 91 | 4.50521E-05 | 79 | 111 | accept | 0.778620616 |
| 1月9日 | 2088048 | 69 | 4.50521E-05 | 78 | 110 | reject | 0.008297825 |
| 1月5日 | 2025467 | 81 | 4.50521E-05 | 76 | 107 | accept | 0.306737318 |
| 1月2日 | 1883075 | 90 | 4.50521E-05 | 70 | 100 | accept | 0.531204805 |
| 12月29日 | 3156543 | 156 | 4.50521E-05 | 123 | 162 | accept | 0.232900504 |
| 12月26日 | 1956325 | 129 | 4.50521E-05 | 73 | 104 | reject | 3.59799E-05 |
| 12月22日 | 1848658 | 61 | 4.50521E-05 | 69 | 99 | reject | 0.012938894 |
| 12月19日 | 1924018 | 122 | 4.50521E-05 | 72 | 102 | reject | 0.00027628 |



is the number of winning fourth prize in a draw. is the total trials in each draw. The following parameters can be derived as:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  |  |  | CL | UCL | LCL |
| 4.41845E-05 | 2805962.42 | 3.96812E-06 | 4.42E-05 | 5.61E-05 | 3.23E-05 |

|  |  |  |
| --- | --- | --- |
|  |  |  |
| Binomial.dist | 0.001850711 | 0.000831981 |



In the beginning, there has been the special number in a set. As a result, there are only five numbers we can choose. The outcome space is . Four number should be the winning numbers. The rest of it should not be any winning number. The probability in this situation is:



The probability of typeⅡerror is . is the trial number in each draw. is the shifted probability. and is the upper bound and lower bound constructed in (iii) respectively. And the following parameter is:

the result:

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| 開獎日 | 注數 | 4獎中獎數 | p\_0 | Binom\_0.05(lb) | Binom\_0.95(ub) | p\_1 | P\_new(lb) | P\_new(ub) | β |
| 4月23日 | 1890662 | 78 | 4.50521E-05 | 70 | 101 | 0.0004 | 6.1E-204 | 1.07E-175 | 1.1E-175 |
| 4月19日 | 1825215 | 83 | 4.50521E-05 | 68 | 97 | 0.0004 | 1.6E-196 | 4.21E-170 | 4.2E-170 |
| 4月16日 | 1875931 | 80 | 4.50521E-05 | 70 | 100 | 0.0004 | 8E-202 | 1.6E-174 | 1.6E-174 |
| 4月12日 | 1898399 | 89 | 4.50521E-05 | 71 | 101 | 0.0004 | 4.6E-204 | 9.34E-177 | 9.3E-177 |
| 4月9日 | 2217157 | 139 | 4.50521E-05 | 84 | 117 | 0.0004 | 5.9E-237 | 6.28E-207 | 6.3E-207 |
| 4月9日 | 2023176 | 76 | 4.50521E-05 | 76 | 107 | 0.0004 | 5E-217 | 8.66E-189 | 8.7E-189 |
| 4月2日 | 2044755 | 77 | 4.50521E-05 | 77 | 108 | 0.0004 | 3.9E-219 | 6.68E-191 | 6.7E-191 |
| 3月29日 | 1962803 | 93 | 4.50521E-05 | 73 | 104 | 0.0004 | 2.5E-211 | 4.77E-183 | 4.8E-183 |
| 3月26日 | 1973645 | 87 | 4.50521E-05 | 74 | 105 | 0.0004 | 6.8E-212 | 1.08E-183 | 1.1E-183 |
| 3月22日 | 2018177 | 90 | 4.50521E-05 | 76 | 107 | 0.0004 | 2.6E-216 | 4.19E-188 | 4.2E-188 |
| 3月19日 | 2057045 | 111 | 4.50521E-05 | 77 | 109 | 0.0004 | 6.7E-221 | 9.64E-192 | 9.6E-192 |
| 3月15日 | 2052515 | 100 | 4.50521E-05 | 77 | 109 | 0.0004 | 3E-220 | 4.02E-191 | 4E-191 |
| 3月12日 | 2945834 | 100 | 4.50521E-05 | 114 | 152 | 0.0004 | 0 | 3.09E-277 | 3.1E-277 |
| 3月8日 | 2651765 | 100 | 4.50521E-05 | 102 | 138 | 0.0004 | 1.6E-281 | 9.36E-249 | 9.4E-249 |
| 3月5日 | 2693112 | 143 | 4.50521E-05 | 104 | 140 | 0.0004 | 1.7E-285 | 9.86E-253 | 9.9E-253 |
| 3月1日 | 2397648 | 89 | 4.50521E-05 | 91 | 125 | 0.0004 | 6.2E-256 | 6.57E-225 | 6.6E-225 |
| 2月27日 | 2285357 | 107 | 4.50521E-05 | 87 | 120 | 0.0004 | 8.9E-244 | 9.66E-214 | 9.7E-214 |
| 2月24日 | 2814840 | 150 | 4.50521E-05 | 109 | 146 | 0.0004 | 5E-298 | 2.39E-264 | 2.4E-264 |
| 2月23日 | 3095869 | 153 | 4.50521E-05 | 120 | 159 | 0.0004 | 0 | 6.86E-292 | 6.9E-292 |
| 2月22日 | 2739793 | 127 | 4.50521E-05 | 105 | 142 | 0.0004 | 3.4E-291 | 1.92E-257 | 1.9E-257 |
| 2月21日 | 2772715 | 119 | 4.50521E-05 | 107 | 144 | 0.0004 | 6E-294 | 2.9E-260 | 2.9E-260 |
| 2月20日 | 3122054 | 146 | 4.50521E-05 | 121 | 160 | 0.0004 | 0 | 1.23E-294 | 1.2E-294 |
| 2月19日 | 2963152 | 133 | 4.50521E-05 | 115 | 153 | 0.0004 | 0 | 9.17E-279 | 9.2E-279 |
| 2月18日 | 3034165 | 120 | 4.50521E-05 | 118 | 156 | 0.0004 | 0 | 5.75E-286 | 5.7E-286 |
| 2月17日 | 3296659 | 148 | 4.50521E-05 | 129 | 169 | 0.0004 | 0 | 0 | 0 |
| 2月16日 | 3766045 | 151 | 4.50521E-05 | 149 | 191 | 0.0004 | 0 | 0 | 0 |
| 2月15日 | 3946720 | 187 | 4.50521E-05 | 156 | 200 | 0.0004 | 0 | 0 | 0 |
| 2月14日 | 5440029 | 233 | 4.50521E-05 | 220 | 271 | 0.0004 | 0 | 0 | 0 |
| 2月13日 | 3807036 | 130 | 4.50521E-05 | 150 | 193 | 0.0004 | 0 | 0 | 0 |
| 2月12日 | 4978061 | 190 | 4.50521E-05 | 200 | 249 | 0.0004 | 0 | 0 | 0 |
| 2月11日 | 5421670 | 178 | 4.50521E-05 | 219 | 270 | 0.0004 | 0 | 0 | 0 |
| 2月10日 | 6228404 | 344 | 4.50521E-05 | 253 | 308 | 0.0004 | 0 | 0 | 0 |
| 2月9日 | 5782611 | 271 | 4.50521E-05 | 234 | 287 | 0.0004 | 0 | 0 | 0 |
| 2月8日 | 3673079 | 158 | 4.50521E-05 | 145 | 187 | 0.0004 | 0 | 0 | 0 |
| 2月7日 | 2942186 | 109 | 4.50521E-05 | 114 | 152 | 0.0004 | 0 | 9.81E-277 | 9.8E-277 |
| 2月6日 | 4308773 | 216 | 4.50521E-05 | 171 | 217 | 0.0004 | 0 | 0 | 0 |
| 2月2日 | 2179897 | 81 | 4.50521E-05 | 82 | 115 | 0.0004 | 1.4E-233 | 1.64E-203 | 1.6E-203 |
| 1月30日 | 2135837 | 78 | 4.50521E-05 | 80 | 113 | 0.0004 | 3.1E-229 | 3.66E-199 | 3.7E-199 |
| 1月26日 | 2048380 | 69 | 4.50521E-05 | 77 | 108 | 0.0004 | 1.2E-219 | 2.13E-191 | 2.1E-191 |
| 1月23日 | 1859946 | 105 | 4.50521E-05 | 69 | 99 | 0.0004 | 1.6E-200 | 3.56E-173 | 3.6E-173 |
| 1月19日 | 2058135 | 72 | 4.50521E-05 | 77 | 109 | 0.0004 | 4.7E-221 | 6.84E-192 | 6.8E-192 |
| 1月16日 | 2091740 | 90 | 4.50521E-05 | 79 | 110 | 0.0004 | 6.7E-224 | 1.19E-195 | 1.2E-195 |
| 1月12日 | 2094995 | 91 | 4.50521E-05 | 79 | 111 | 0.0004 | 2.3E-224 | 2.96E-195 | 3E-195 |
| 1月9日 | 2088048 | 69 | 4.50521E-05 | 78 | 110 | 0.0004 | 2.3E-224 | 3.81E-195 | 3.8E-195 |
| 1月5日 | 2025467 | 81 | 4.50521E-05 | 76 | 107 | 0.0004 | 2.4E-217 | 4.21E-189 | 4.2E-189 |
| 1月2日 | 1883075 | 90 | 4.50521E-05 | 70 | 100 | 0.0004 | 7.5E-203 | 1.69E-175 | 1.7E-175 |
| 12月29日 | 3156543 | 156 | 4.50521E-05 | 123 | 162 | 0.0004 | 0 | 1.13E-297 | 1.1E-297 |
| 12月26日 | 1956325 | 129 | 4.50521E-05 | 73 | 104 | 0.0004 | 2.1E-210 | 3.67E-182 | 3.7E-182 |
| 12月22日 | 1848658 | 61 | 4.50521E-05 | 69 | 99 | 0.0004 | 6.7E-199 | 1.24E-171 | 1.2E-171 |
| 12月19日 | 1924018 | 122 | 4.50521E-05 | 72 | 102 | 0.0004 | 9.5E-207 | 2.02E-179 | 2E-179 |



The probability of typeⅡerror is . is the trial number in each draw. is the shifted probability. And the following parameter is:

the result:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  | ) | β |  |
| 0.000368 | 5.7171E-254 | 0 | 5.7E-254 | 1 |



The accident death per month was calculated by summing up the death number in each city in a month. The number is 84 because there are 84 months from Jan.2011 to Dec.2017. The maximum death in a month is 139 and the minimum death is 64. As a result, 78 kinds of death number (~63,64,…,139,140~) had been tested. The Poisson parameter is 10.

Death in a month = sum of number of deaths from New Taipei, Taipei, …, Keelung

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| **n** | | **max death number** | | **min death number** | | | **k** |
| 84 | | 139 | | 64 | | | 78 |
| **death number** | **x\_i** | | **frequence** | | **p\_i(poisson, λ=10)** | | **np\_i** | **(X\_i-np\_i)^2/np\_i** | | |
| ~63 | 0 | | 0 | | 1 | | 84 | 84 | | |
| 64 | 2 | | 0.02381 | | 3.578E-30 | | 3.01E-28 | 1.33089E+28 | | |
| 65 | 0 | | 0 | | 5.505E-31 | | 4.62E-29 | 4.62386E-29 | | |
| 66 | 0 | | 0 | | 8.34E-32 | | 7.01E-30 | 7.00584E-30 | | |
| 67 | 0 | | 0 | | 1.245E-32 | | 1.05E-30 | 1.04565E-30 | | |
| 68 | 1 | | 0.011905 | | 1.831E-33 | | 1.54E-31 | 6.50315E+30 | | |
| 69 | 0 | | 0 | | 2.653E-34 | | 2.23E-32 | 2.22858E-32 | | |
| 70 | 2 | | 0.02381 | | 3.79E-35 | | 3.18E-33 | 1.25641E+33 | | |
| 71 | 0 | | 0 | | 5.338E-36 | | 4.48E-34 | 4.48406E-34 | | |
| 72 | 1 | | 0.011905 | | 7.414E-37 | | 6.23E-35 | 1.60569E+34 | | |
| 73 | 0 | | 0 | | 1.016E-37 | | 8.53E-36 | 8.53131E-36 | | |
| 74 | 4 | | 0.047619 | | 1.372E-38 | | 1.15E-36 | 1.38783E+37 | | |
| 75 | 1 | | 0.011905 | | 1.83E-39 | | 1.54E-37 | 6.50545E+36 | | |
| 76 | 1 | | 0.011905 | | 2.408E-40 | | 2.02E-38 | 4.94414E+37 | | |
| 77 | 2 | | 0.02381 | | 3.127E-41 | | 2.63E-39 | 1.5228E+39 | | |
| 78 | 0 | | 0 | | 4.009E-42 | | 3.37E-40 | 3.36763E-40 | | |
| 79 | 7 | | 0.083333 | | 5.075E-43 | | 4.26E-41 | 1.14947E+42 | | |
| 80 | 3 | | 0.035714 | | 6.343E-44 | | 5.33E-42 | 1.68902E+42 | | |
| 81 | 2 | | 0.02381 | | 7.831E-45 | | 6.58E-43 | 6.08049E+42 | | |
| 82 | 1 | | 0.011905 | | 9.551E-46 | | 8.02E-44 | 1.2465E+43 | | |
| 83 | 1 | | 0.011905 | | 1.151E-46 | | 9.67E-45 | 1.03459E+44 | | |
| 84 | 2 | | 0.02381 | | 1.37E-47 | | 1.15E-45 | 3.47624E+45 | | |
| 85 | 1 | | 0.011905 | | 1.612E-48 | | 1.35E-46 | 7.38701E+45 | | |
| 86 | 0 | | 0 | | 1.874E-49 | | 1.57E-47 | 1.5741E-47 | | |
| 87 | 3 | | 0.035714 | | 2.154E-50 | | 1.81E-48 | 4.97426E+48 | | |
| 88 | 0 | | 0 | | 2.448E-51 | | 2.06E-49 | 2.05604E-49 | | |
| 89 | 0 | | 0 | | 2.75E-52 | | 2.31E-50 | 2.31016E-50 | | |
| 90 | 0 | | 0 | | 3.056E-53 | | 2.57E-51 | 2.56684E-51 | | |
| 91 | 1 | | 0.011905 | | 3.358E-54 | | 2.82E-52 | 3.54522E+51 | | |
| 92 | 4 | | 0.047619 | | 3.65E-55 | | 3.07E-53 | 5.21856E+53 | | |
| 93 | 3 | | 0.035714 | | 3.925E-56 | | 3.3E-54 | 2.72996E+54 | | |
| 94 | 5 | | 0.059524 | | 4.175E-57 | | 3.51E-55 | 7.12822E+55 | | |
| 95 | 0 | | 0 | | 4.395E-58 | | 3.69E-56 | 3.69177E-56 | | |
| 96 | 2 | | 0.02381 | | 4.578E-59 | | 3.85E-57 | 1.04015E+57 | | |
| 97 | 1 | | 0.011905 | | 4.72E-60 | | 3.96E-58 | 2.52236E+57 | | |
| 98 | 0 | | 0 | | 4.816E-61 | | 4.05E-59 | 4.04544E-59 | | |
| 99 | 3 | | 0.035714 | | 4.865E-62 | | 4.09E-60 | 2.20248E+60 | | |
| 100 | 1 | | 0.011905 | | 4.865E-63 | | 4.09E-61 | 2.4472E+60 | | |
| 101 | 1 | | 0.011905 | | 4.816E-64 | | 4.05E-62 | 2.47167E+61 | | |
| 102 | 1 | | 0.011905 | | 4.722E-65 | | 3.97E-63 | 2.5211E+62 | | |
| 103 | 1 | | 0.011905 | | 4.585E-66 | | 3.85E-64 | 2.59674E+63 | | |
| 104 | 2 | | 0.02381 | | 4.408E-67 | | 3.7E-65 | 1.08024E+65 | | |
| 105 | 1 | | 0.011905 | | 4.198E-68 | | 3.53E-66 | 2.83564E+65 | | |
| 106 | 2 | | 0.02381 | | 3.961E-69 | | 3.33E-67 | 1.20231E+67 | | |
| 107 | 2 | | 0.02381 | | 3.702E-70 | | 3.11E-68 | 1.28647E+68 | | |
| 108 | 2 | | 0.02381 | | 3.427E-71 | | 2.88E-69 | 1.38939E+69 | | |
| 109 | 1 | | 0.011905 | | 3.144E-72 | | 2.64E-70 | 3.78609E+69 | | |
| 110 | 0 | | 0 | | 2.858E-73 | | 2.4E-71 | 2.40114E-71 | | |
| 111 | 3 | | 0.035714 | | 2.575E-74 | | 2.16E-72 | 4.16053E+72 | | |
| 112 | 1 | | 0.011905 | | 2.299E-75 | | 1.93E-73 | 5.17755E+72 | | |
| 113 | 2 | | 0.02381 | | 2.035E-76 | | 1.71E-74 | 2.34025E+74 | | |
| 114 | 0 | | 0 | | 1.785E-77 | | 1.5E-75 | 1.49931E-75 | | |
| 115 | 2 | | 0.02381 | | 1.552E-78 | | 1.3E-76 | 3.06807E+76 | | |
| 116 | 0 | | 0 | | 1.338E-79 | | 1.12E-77 | 1.12392E-77 | | |
| 117 | 0 | | 0 | | 1.144E-80 | | 9.61E-79 | 9.60618E-79 | | |
| 118 | 0 | | 0 | | 9.691E-82 | | 8.14E-80 | 8.14083E-80 | | |
| 119 | 1 | | 0.011905 | | 8.144E-83 | | 6.84E-81 | 1.46177E+80 | | |
| 120 | 2 | | 0.02381 | | 6.787E-84 | | 5.7E-82 | 7.01648E+81 | | |
| 121 | 2 | | 0.02381 | | 5.609E-85 | | 4.71E-83 | 8.48994E+82 | | |
| 122 | 0 | | 0 | | 4.597E-86 | | 3.86E-84 | 3.86185E-84 | | |
| 123 | 1 | | 0.011905 | | 3.738E-87 | | 3.14E-85 | 3.185E+84 | | |
| 124 | 0 | | 0 | | 3.014E-88 | | 2.53E-86 | 2.53203E-86 | | |
| 125 | 0 | | 0 | | 2.411E-89 | | 2.03E-87 | 2.02562E-87 | | |
| 126 | 0 | | 0 | | 1.914E-90 | | 1.61E-88 | 1.60764E-88 | | |
| 127 | 0 | | 0 | | 1.507E-91 | | 1.27E-89 | 1.26586E-89 | | |
| 128 | 0 | | 0 | | 1.177E-92 | | 9.89E-91 | 9.8895E-91 | | |
| 129 | 0 | | 0 | | 9.127E-94 | | 7.67E-92 | 7.66628E-92 | | |
| 130 | 0 | | 0 | | 7.02E-95 | | 5.9E-93 | 5.89714E-93 | | |
| 131 | 0 | | 0 | | 5.359E-96 | | 4.5E-94 | 4.50163E-94 | | |
| 132 | 0 | | 0 | | 4.06E-97 | | 3.41E-95 | 3.41033E-95 | | |
| 133 | 1 | | 0.011905 | | 3.053E-98 | | 2.56E-96 | 3.89992E+95 | | |
| 134 | 0 | | 0 | | 2.28E-99 | | 1.91E-97 | 1.91355E-97 | | |
| 135 | 0 | | 0 | | 1.69E-100 | | 1.42E-98 | 1.41744E-98 | | |
| 136 | 0 | | 0 | | 1.24E-101 | | 1E-99 | 1.0422E-99 | | |
| 137 | 0 | | 0 | | 9.06E-103 | | 7.6E-101 | 7.6076E-101 | | |
| 138 | 1 | | 0.011905 | | 6.56E-104 | | 5.5E-102 | 1.814E+101 | | |
| 139 | 1 | | 0.011905 | | 4.72E-105 | | 4E-103 | 2.5214E+102 | | |
| 140~ | 0 | | 0 | | 0 | | 0 | infinity | | |

The result shows that the accident death per month over the period didn’t follow a Poisson distribution with .



Suppose the accident death per month in Taoyuan followed a Poisson distribution. is the probability of x in Poisson distribution. The upper bound and lower bound in the Hypothesis test with can be constructed by the following test:

Since Poisson is a discrete distribution, the upper bound and lower bound in the Hypothesis test with are not really accurate. I use the p-value to do the Hypothesis test. If the p-value is smaller than α=0.05, the death number in that month reject H0.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| month | death in Taoyuan | p-value | H\_0 | month | death in Taoyuan | p-value | H\_0 |
| 1-Jan | 7 | 0.4404413 | accept | 1-Mar | 18 | 0.01437301 | reject |
| 1-Feb | 7 | 0.4404413 | accept | 1-Apr | 16 | 0.05408322 | accept |
| 1-Mar | 5 | 0.1341719 | accept | 1-May | 18 | 0.01437301 | reject |
| 1-Apr | 4 | 0.0585054 | accept | 1-Jun | 7 | 0.44044129 | accept |
| 1-May | 9 | 0.9158594 | accept | 1-Jul | 15 | 0.09748081 | accept |
| 1-Jun | 7 | 0.4404413 | accept | 1-Aug | 14 | 0.16691695 | accept |
| 1-Jul | 10 | 0.8339205 | accept | 1-Sep | 15 | 0.09748081 | accept |
| 1-Aug | 6 | 0.2602828 | accept | 1-Oct | 13 | 0.27107115 | accept |
| 1-Sep | 14 | 0.1669169 | accept | 1-Nov | 18 | 0.01437301 | reject |
| 1-Oct | 6 | 0.2602828 | accept | 1-Dec | 22 | 0.00059147 | reject |
| 1-Nov | 13 | 0.2710712 | accept | 1-Jan | 18 | 0.01437301 | reject |
| 1-Dec | 5 | 0.1341719 | accept | 1-Feb | 10 | 0.8339205 | accept |
| 1-Jan | 21 | 0.0013993 | reject | 1-Mar | 24 | 9.3899E-05 | reject |
| 1-Feb | 13 | 0.2710712 | accept | 1-Apr | 13 | 0.27107115 | accept |
| 1-Mar | 13 | 0.2710712 | accept | 1-May | 12 | 0.41688705 | accept |
| 1-Apr | 11 | 0.6064477 | accept | 1-Jun | 13 | 0.27107115 | accept |
| 1-May | 22 | 0.0005915 | reject | 1-Jul | 19 | 0.00690868 | reject |
| 1-Jun | 17 | 0.0285552 | reject | 1-Aug | 9 | 0.91585943 | accept |
| 1-Jul | 9 | 0.9158594 | accept | 1-Sep | 15 | 0.09748081 | accept |
| 1-Aug | 16 | 0.0540832 | accept | 1-Oct | 24 | 9.3899E-05 | reject |
| 1-Sep | 22 | 0.0005915 | reject | 1-Nov | 14 | 0.16691695 | accept |
| 1-Oct | 16 | 0.0540832 | accept | 1-Dec | 12 | 0.41688705 | accept |
| 1-Nov | 11 | 0.6064477 | accept | 1-Jan | 18 | 0.01437301 | reject |
| 1-Dec | 17 | 0.0285552 | reject | 1-Feb | 15 | 0.09748081 | accept |
| 1-Jan | 21 | 0.0013993 | reject | 1-Mar | 23 | 0.00024024 | reject |
| 1-Feb | 24 | 9.39E-05 | reject |  |  |  |  |



The upper bound in the Hypothesis test is 17, and the lower bound is 4. The type Пerror probability is in Poisson distribution with .



The parameter is the total death in all cities per month. The parameter because there are totally 84 months from Jan.2011 to Dec.2017. The is estimated by the equation:

The standard error is the square root of . Finally, CL, UCL, and LCL can be constructed:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **σ** | **CL** | **UCL** | **LCL** |
| 94.54762 | 9.72356 | 94.54762 | 123.7183 | 65.37694 |

Use the three line to monitor the number of accident deaths from Jan.2018 to March 2022.

The probability of typeⅠerror is:

where is the probability of x in Poisson with . And ].

|  |  |  |  |
| --- | --- | --- | --- |
|  |  | α |  |
| 0.002128 | 0.000828 | 0.002956 | 338.3434 |



The average thicknesses and the standard error of right side and left side could be calculated by the 85-wafers sample in bottom zone.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| average | | | | |
| up | middle | down | left | right |
| 349.7176 | 349.6471 | 356.1882 | 348.7412 | 357.0941 |
| sample variance | | | | |
| 23.75266 | 15.15966 | 142.1308 | 138.1941 | 32.56246 |
| standard error | | | | |
| 4.87367 | 3.893541 | 11.92186 | 11.7556 | 5.706353 |

The t-distribution must be used in this Hypothesis test. The DOF in t-distribution is because there are 85 numbers in a sample. The is 0.02. Consequently, the result will reject if or .



The proportion between two sample variances might be used in the Hypothesis test. The smaller on should be the denominator. F-test also must be used to test whether the result reject or not. The result will accept if .



The maximum and minimum are considered as extreme values in those 85 numbers. Therefore, only the values between the second biggest number and the second smallest number are counted precisely in the histogram.

The data in the Q-Q plot has been rearranged from the smallest to the largest. The {x , y} in Q-Q plot is {, th smallest sample observation}. The Gamma distribution has been assumed in this data. The parameter of Gamma can be estimated by following:

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| average | sample variance | standard error | estimated α | estimated β |
| 348.7412 | 138.1941 | 11.7556 | 880.0694 | 0.396266 |

To do the proportion test, the appearance numbers of 27 kinds thickness value counted in the histogram must be compared with the probability of those value in Gamma distribution.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| thickness | number x\_i | p\_i (gamma.dist) | np\_i | thickness | number x\_i | p\_i (gamma.dist) | np\_i |
| <334 | 1 | 0.087913 | 7.472574 | 347 | 5 | 0.03373 | 2.867063 |
| 334 | 1 | 0.015776 | 1.340944 | 348 | 2 | 0.033938 | 2.884713 |
| 335 | 1 | 0.017513 | 1.488584 | 349 | 10 | 0.0339 | 2.88148 |
| 336 | 0 | 0.019289 | 1.639587 | 350 | 6 | 0.033618 | 2.857553 |
| 337 | 0 | 0.021081 | 1.791899 | 351 | 12 | 0.033101 | 2.813561 |
| 338 | 0 | 0.022862 | 1.943262 | 352 | 9 | 0.032359 | 2.75055 |
| 339 | 0 | 0.024603 | 2.091256 | 353 | 5 | 0.031411 | 2.669941 |
| 340 | 1 | 0.026275 | 2.233372 | 354 | 4 | 0.030276 | 2.573474 |
| 341 | 1 | 0.027848 | 2.367076 | 355 | 2 | 0.028978 | 2.463154 |
| 342 | 0 | 0.029293 | 2.48989 | 356 | 4 | 0.027543 | 2.341175 |
| 343 | 2 | 0.030582 | 2.599466 | 357 | 4 | 0.025998 | 2.209855 |
| 344 | 5 | 0.03169 | 2.693662 | 358 | 1 | 0.024371 | 2.071564 |
| 345 | 4 | 0.032595 | 2.770613 | >358 | 1 | 0.214193 | 18.20639 |
| 346 | 4 | 0.03328 | 2.828792 | k=27 |  |  |  |

The result shows that the thickness wasn’t follow the Gamma distribution.



Sample mean and standard error are estimated from first 45 wafers.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | sample variance | standard error σ | CL | UCL | LCL |
| 354.3911 | 26.34174 | 5.13242 | 354.3911 | 369.7884 | 338.9939 |

3 thickness values exceeded the boundary of  chart.

Sample mean of range and standard error are estimated from first 45 wafers.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | sample variance | standard error σ | CL | UCL | LCL |
| 9.822222 | 6.740404 | 2.596229 | 9.822222 | 17.61091 | 2.033536 |

One R value exceeded the boundary of chart.

Remove the extreme wafers in the previous  chart and chart, and construct a chart and chart again with the rest of 41 wafers.

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | sample variance | standard error σ | CL | UCL | LCL |
| 354.9707 | 6.561122 | 2.561469 | 354.9707 | 362.6551 | 347.2863 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | sample variance | standard error σ | CL | UCL | LCL |
| 9.902439 | 5.290244 | 2.300053 | 9.902439 | 16.8026 | 3.00228 |

The result shows that there isn’t any value exceed the boundary in chart and chart. The same CL, UCL, and LCL in both charts can be used to construct the Shewhart control chart to monitor the last 40 wafers.

It is obvious that there is a wafer out of range in the both charts. Most of the wafers’ are lower than the control line, and some wafers’ are lower than the control bound in chart.



In the sequential likelihood ratio test with , is 350, standard error could be estimated by , is , and could be 1 to 40. must be constructed by the following equation. will be reset to zero if exceed the boundaries of sequential likelihood ratio test.

|  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| average | sample variance | standard error σ |  |  | Δ | α | β | accept (if lower) | accept (if higher) |
| 354.3911 | 26.34174 | 5.13242 | 350 | 341.7881 | -8.21187 | 0.003 | 0.2 | -17.9186 | 5.153057 |

Sequential likelihood ratio test shows that most of the tend to close to . It accepts and rejects .

In the “graphical” Tabular CUSUM chart, . and can be constructed by the following equation:

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| average | sample variance | standard error σ |  | k | K | h | H |
| 354.3911 | 26.34174 | 5.13242 | 350 | 0.8 | 4.105936 | 6 | 30.79452 |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| average | sample variance | standard error σ |  | k | K | h | H |
| 354.3911 | 26.34174 | 5.13242 | 350 | 0.5 | 2.56621 | 5 | 25.6621 |

There is an out -of-control signal in Tabular CUSUM chart with . The new process mean is :



|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| shift | δ | k\* | h' |  |
| 0.5σ | 0.5 | 0.8 | 4.475606 | 60.87891 |
| 1.0σ | 1 | 0.8 | 4.475606 | 11.96443 |
| 1.5σ | 1.5 | 0.8 | 4.475606 | 5.375253 |
| 2.0σ | 2 | 0.8 | 4.475606 | 3.382457 |

There is an out -of-control signal in optimal Tabular CUSUM chart. The new process mean is :



|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| shift | δ | λ\* | L\* | g | w | φ(w) | Φ(w) |  |
| 0.5σ | 0.5 | 0.232431 | 3.12491 | 2.130589 | 2.055716 | 0.048223 | 0.980095 | 42.53612 |
| 1.0σ | 1 | 0.232431 | 3.12491 | 1.065295 | 0.753675 | 0.300306 | 0.774478 | 14.72196 |
| 1.5σ | 1.5 | 0.232431 | 3.12491 | 0.710196 | -0.58667 | 0.33587 | 0.278711 | 5.806391 |
| 2.0σ | 2 | 0.232431 | 3.12491 | 0.532647 | -1.94685 | 0.059961 | 0.025776 | 3.95144 |



|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | sample variance | standard error |  | USL | LSL |  |  |
| 352.2776 | 350 | 82.62084 | 9.089601 | 9.37062 | 360 | 335 | 0.849581 | -1.90081 |

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  | out-of-spec% |
| 0.458399 | 0.283194 | 0.444652 | 0.355722 | 22.64% |



|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| wafer number | X\_bar | σ\_X | σ^~ | Z\_U | Z\_L |
| 1 | 354.6 | 3.209361 | 5.608921 | 1.682578 | -6.10713 |
| 2 | 353.8 | 3.114482 | 4.913247 | 1.9907 | -6.03632 |
| 3 | 353.6 | 3.130495 | 4.770744 | 2.044405 | -5.94155 |
| 4 | 357.6 | 3.847077 | 8.518216 | 0.62385 | -5.87459 |
| 5 | 353.8 | 2.774887 | 4.705316 | 2.234325 | -6.77505 |
| 6 | 352.4 | 2.19089 | 3.249615 | 3.46891 | -7.94198 |
| 7 | 355.6 | 5.029911 | 7.527284 | 0.874767 | -4.0955 |
| 8 | 353.2 | 3.563706 | 4.789572 | 1.908126 | -5.10704 |
| 9 | 355 | 3.24037 | 5.958188 | 1.543033 | -6.17213 |
| 10 | 354.4 | 3.435113 | 5.582114 | 1.630223 | -5.64756 |
| 11 | 354 | 3 | 5 | 2 | -6.33333 |
| 12 | 353.2 | 4.207137 | 5.28583 | 1.616301 | -4.32598 |
| 13 | 354.2 | 5.357238 | 6.807349 | 1.082647 | -3.58394 |
| 14 | 353.8 | 4.868265 | 6.175759 | 1.273554 | -3.86175 |
| 15 | 354.6 | 4.929503 | 6.742403 | 1.095445 | -3.97606 |
| 16 | 353.2 | 4.494441 | 5.517246 | 1.51298 | -4.04945 |
| 17 | 370.4 | 3.507136 | 20.69928 | -2.96538 | -10.0937 |
| 18 | 357.2 | 4.147288 | 8.309031 | 0.67514 | -5.3529 |
| 19 | 359.4 | 6.0663 | 11.18749 | 0.098907 | -4.02222 |
| 20 | 356.6 | 5.412947 | 8.535807 | 0.628124 | -3.99043 |
| 21 | 357.8 | 5.674504 | 9.645724 | 0.387699 | -4.01797 |
| 22 | 356.8 | 3.03315 | 7.445804 | 1.055009 | -7.18725 |
| 23 | 360.6 | 4.97996 | 11.71153 | -0.12048 | -5.1406 |
| 24 | 360.2 | 4.969909 | 11.34637 | -0.04024 | -5.07051 |
| 25 | 358.8 | 5.357238 | 10.30243 | 0.223996 | -4.44259 |
| 26 | 338 | 4.949747 | 12.98075 | 4.444671 | -0.60609 |
| 27 | 354.6 | 5.899152 | 7.480642 | 0.915386 | -3.32251 |
| 28 | 349.2 | 0.83666 | 1.157584 | 12.90847 | -16.9722 |
| 29 | 356.6 | 5.549775 | 8.623224 | 0.612637 | -3.89205 |
| 30 | 357 | 5.09902 | 8.660254 | 0.588348 | -4.31455 |
| 31 | 356.8 | 5.761944 | 8.912912 | 0.555368 | -3.78345 |
| 32 | 355.4 | 6.580274 | 8.512344 | 0.699059 | -3.10018 |
| 33 | 354.8 | 6.180615 | 7.825599 | 0.84134 | -3.20356 |
| 34 | 352.6 | 6.426508 | 6.932532 | 1.151481 | -2.73866 |
| 35 | 351 | 5.244044 | 5.338539 | 1.716233 | -3.05108 |
| 36 | 351.2 | 1.923538 | 2.267157 | 4.574902 | -8.42198 |
| 37 | 348 | 4.795832 | 5.196152 | 2.502173 | -2.71069 |
| 38 | 357.8 | 4.816638 | 9.167333 | 0.45675 | -4.73359 |
| 39 | 354.6 | 5.458938 | 7.138627 | 0.989203 | -3.59044 |
| 40 | 352.4 | 3.286335 | 4.069398 | 2.312606 | -5.29465 |
| 41 | 354.6 | 4.97996 | 6.779381 | 1.084346 | -3.93577 |
| 42 | 336.2 | 4.816638 | 14.61643 | 4.941206 | -0.24914 |
| 43 | 352.6 | 5.412947 | 6.004998 | 1.367093 | -3.25146 |
| 44 | 356 | 5.522681 | 8.154753 | 0.724286 | -3.8025 |
| 45 | 353.4 | 4.27785 | 5.46443 | 1.542831 | -4.30123 |
| 46 | 355.2 | 4.868265 | 7.123202 | 0.985978 | -4.14932 |
| 47 | 355.6 | 4.560702 | 7.222188 | 0.964764 | -4.51685 |
| 48 | 317.6 | 55.55448 | 64.31221 | 0.763215 | 0.313206 |
| 49 | 359 | 4.582576 | 10.0995 | 0.218218 | -5.23723 |
| 50 | 354 | 4.582576 | 6.082763 | 1.309307 | -4.14614 |
| 51 | 354.8 | 4.764452 | 6.763135 | 1.091416 | -4.15578 |
| 52 | 357 | 4.690416 | 8.42615 | 0.639602 | -4.69042 |
| 53 | 351.8 | 5.263079 | 5.562374 | 1.558023 | -3.19205 |
| 54 | 353.4 | 5.128353 | 6.153048 | 1.286963 | -3.5879 |
| 55 | 352.2 | 4.868265 | 5.342284 | 1.602214 | -3.53309 |
| 56 | 348.6 | 1.516575 | 2.063977 | 7.516937 | -8.96757 |
| 57 | 349 | 1.581139 | 1.870829 | 6.957011 | -8.85438 |
| 58 | 349.4 | 5.128353 | 5.163332 | 2.066941 | -2.80792 |
| 59 | 349.4 | 6.14817 | 6.177378 | 1.72409 | -2.34216 |
| 60 | 352.6 | 5.029911 | 5.662155 | 1.471199 | -3.49907 |
| 61 | 353.2 | 4.32435 | 5.379591 | 1.572491 | -4.20873 |
| 62 | 353 | 3.674235 | 4.743416 | 1.905159 | -4.89898 |
| 63 | 350 | 5.244044 | 5.244044 | 1.906925 | -2.86039 |
| 64 | 351.8 | 5.263079 | 5.562374 | 1.558023 | -3.19205 |
| 65 | 352.4 | 5.176872 | 5.706137 | 1.468068 | -3.3611 |
| 66 | 344.2 | 2.683282 | 6.390618 | 5.888312 | -3.42864 |
| 67 | 350.4 | 4.722288 | 4.739198 | 2.032913 | -3.26113 |
| 68 | 349.2 | 4.494441 | 4.565085 | 2.402968 | -3.15946 |
| 69 | 348.2 | 4.38178 | 4.737088 | 2.692969 | -3.01247 |
| 70 | 348.6 | 4.09878 | 4.331282 | 2.781315 | -3.31806 |
| 71 | 350.8 | 4.868265 | 4.933559 | 1.88979 | -3.24551 |
| 72 | 343.8 | 3.49285 | 7.116179 | 4.638047 | -2.51943 |
| 73 | 349.4 | 5.176872 | 5.211526 | 2.047569 | -2.7816 |
| 74 | 348.2 | 4.91935 | 5.23832 | 2.398691 | -2.68328 |
| 75 | 346.8 | 5.263079 | 6.159545 | 2.508038 | -2.24203 |
| 76 | 351 | 5.477226 | 5.567764 | 1.643168 | -2.92119 |
| 77 | 346.8 | 4.38178 | 5.425864 | 3.012474 | -2.69297 |
| 78 | 349.6 | 5.458938 | 5.473573 | 1.905133 | -2.67451 |
| 79 | 350.6 | 5.412947 | 5.4461 | 1.736577 | -2.88198 |
| 80 | 351 | 5.612486 | 5.700877 | 1.603567 | -2.85079 |
| 81 | 349.4 | 5.366563 | 5.4 | 1.975193 | -2.68328 |
| 82 | 351.4 | 5.59464 | 5.767148 | 1.537186 | -2.93138 |
| 83 | 345.2 | 1.788854 | 5.122499 | 8.273452 | -5.70197 |
| 84 | 353.4 | 4.722288 | 5.818935 | 1.397628 | -3.89642 |
| 85 | 348 | 4.582576 | 5 | 2.618615 | -2.83683 |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| wafer number | C\_p | C\_pk | C\_pm | C\*\_pm | out-of-spec% |
| 1 | 1.298285 | 0.560859 | 0.742864 | 0.594291 | 0.046228429 |
| 2 | 1.337836 | 0.663567 | 0.848047 | 0.678438 | 0.023256939 |
| 3 | 1.330993 | 0.681468 | 0.873379 | 0.698703 | 0.020456778 |
| 4 | 1.083073 | 0.20795 | 0.489148 | 0.391318 | 0.266362955 |
| 5 | 1.501562 | 0.744775 | 0.885523 | 0.708419 | 0.012730851 |
| 6 | 1.901814 | 1.156303 | 1.282203 | 1.025762 | 0.000261288 |
| 7 | 0.828378 | 0.291589 | 0.553542 | 0.442833 | 0.190871397 |
| 8 | 1.169195 | 0.636042 | 0.869946 | 0.695956 | 0.028187629 |
| 9 | 1.285861 | 0.514344 | 0.699318 | 0.559454 | 0.061411324 |
| 10 | 1.212964 | 0.543408 | 0.746432 | 0.597145 | 0.051527193 |
| 11 | 1.388889 | 0.666667 | 0.833333 | 0.666667 | 0.022750132 |
| 12 | 0.990381 | 0.538767 | 0.788271 | 0.630617 | 0.053022203 |
| 13 | 0.777764 | 0.360882 | 0.612084 | 0.489667 | 0.139651703 |
| 14 | 0.855883 | 0.424518 | 0.674681 | 0.539745 | 0.101466972 |
| 15 | 0.845251 | 0.365148 | 0.617979 | 0.494384 | 0.136695872 |
| 16 | 0.927071 | 0.504327 | 0.755208 | 0.604166 | 0.065168021 |
| 17 | 1.188054 | -0.98846 | 0.201295 | 0.161036 | 0.998488468 |
| 18 | 1.004673 | 0.225047 | 0.501462 | 0.40117 | 0.24979347 |
| 19 | 0.686855 | 0.032969 | 0.37244 | 0.297952 | 0.460634853 |
| 20 | 0.769759 | 0.209375 | 0.48814 | 0.390512 | 0.264994466 |
| 21 | 0.734279 | 0.129233 | 0.43197 | 0.345576 | 0.349148727 |
| 22 | 1.373709 | 0.35167 | 0.559599 | 0.447679 | 0.14571066 |
| 23 | 0.836687 | -0.04016 | 0.355775 | 0.28462 | 0.547949824 |
| 24 | 0.838379 | -0.01341 | 0.367225 | 0.29378 | 0.516050174 |
| 25 | 0.777764 | 0.074665 | 0.404435 | 0.323548 | 0.411384645 |
| 26 | 0.841794 | 0.202031 | 0.320988 | 0.25679 | 0.27223139 |
| 27 | 0.706316 | 0.305129 | 0.556993 | 0.445595 | 0.180440639 |
| 28 | 4.980119 | 4.302823 | 3.599452 | 2.879561 | 6.58965E-65 |
| 29 | 0.750781 | 0.204212 | 0.483191 | 0.386553 | 0.270107737 |
| 30 | 0.817151 | 0.196116 | 0.481125 | 0.3849 | 0.278157227 |
| 31 | 0.723136 | 0.185123 | 0.467487 | 0.373989 | 0.289398787 |
| 32 | 0.633206 | 0.23302 | 0.489485 | 0.391588 | 0.243224585 |
| 33 | 0.674151 | 0.280447 | 0.532441 | 0.425952 | 0.200757364 |
| 34 | 0.648356 | 0.383827 | 0.601031 | 0.480825 | 0.127851784 |
| 35 | 0.794552 | 0.572078 | 0.780488 | 0.624391 | 0.044199832 |
| 36 | 2.166147 | 1.524967 | 1.837838 | 1.47027 | 2.38221E-06 |
| 37 | 0.86881 | 0.834058 | 0.801875 | 0.6415 | 0.009528875 |
| 38 | 0.865057 | 0.15225 | 0.454512 | 0.36361 | 0.323926427 |
| 39 | 0.763274 | 0.329734 | 0.583679 | 0.466943 | 0.161446861 |
| 40 | 1.267876 | 0.770869 | 1.023902 | 0.819122 | 0.010372206 |
| 41 | 0.836687 | 0.361449 | 0.614609 | 0.491687 | 0.139147152 |
| 42 | 0.865057 | 0.083045 | 0.285067 | 0.228054 | 0.40162801 |
| 43 | 0.769759 | 0.455698 | 0.693866 | 0.555093 | 0.086372216 |
| 44 | 0.754465 | 0.241429 | 0.510949 | 0.40876 | 0.234516716 |
| 45 | 0.97401 | 0.514277 | 0.762507 | 0.610006 | 0.061444371 |
| 46 | 0.855883 | 0.328659 | 0.584943 | 0.467954 | 0.162088717 |
| 47 | 0.913602 | 0.321588 | 0.576926 | 0.461541 | 0.167334699 |
| 48 | 0.075001 | -0.1044 | 0.064788 | 0.05183 | 0.845605599 |
| 49 | 0.909241 | 0.072739 | 0.412561 | 0.330049 | 0.413629755 |
| 50 | 0.909241 | 0.436436 | 0.684996 | 0.547997 | 0.095232038 |
| 51 | 0.874532 | 0.363805 | 0.616085 | 0.492868 | 0.137561092 |
| 52 | 0.888336 | 0.213201 | 0.494492 | 0.395594 | 0.261217006 |
| 53 | 0.791679 | 0.519341 | 0.749081 | 0.599265 | 0.060320194 |
| 54 | 0.812477 | 0.428988 | 0.677171 | 0.541737 | 0.09922027 |
| 55 | 0.855883 | 0.534071 | 0.779941 | 0.623953 | 0.054759554 |
| 56 | 2.747419 | 2.505646 | 2.018757 | 1.615005 | 2.80888E-14 |
| 57 | 2.635231 | 2.319004 | 2.227177 | 1.781742 | 1.73783E-12 |
| 58 | 0.812477 | 0.68898 | 0.806972 | 0.645578 | 0.021863013 |
| 59 | 0.677708 | 0.574697 | 0.674504 | 0.539603 | 0.051932029 |
| 60 | 0.828378 | 0.4904 | 0.73588 | 0.588704 | 0.07085208 |
| 61 | 0.963536 | 0.524164 | 0.774532 | 0.619626 | 0.057931228 |
| 62 | 1.134023 | 0.635053 | 0.87841 | 0.702728 | 0.028380205 |
| 63 | 0.794552 | 0.635642 | 0.794552 | 0.635642 | 0.030380755 |
| 64 | 0.791679 | 0.519341 | 0.749081 | 0.599265 | 0.060320194 |
| 65 | 0.804862 | 0.489356 | 0.730208 | 0.584166 | 0.071431025 |
| 66 | 1.552825 | 1.142879 | 0.651997 | 0.521598 | 0.000303311 |
| 67 | 0.882341 | 0.677638 | 0.879192 | 0.703354 | 0.021585498 |
| 68 | 0.927071 | 0.800989 | 0.912725 | 0.73018 | 0.008921608 |
| 69 | 0.950907 | 0.897656 | 0.879584 | 0.703667 | 0.004836578 |
| 70 | 1.016563 | 0.927105 | 0.961994 | 0.769595 | 0.003160182 |
| 71 | 0.855883 | 0.62993 | 0.844556 | 0.675645 | 0.029979197 |
| 72 | 1.192913 | 0.839811 | 0.58552 | 0.468416 | 0.005878964 |
| 73 | 0.804862 | 0.682523 | 0.79951 | 0.639608 | 0.023005704 |
| 74 | 0.846995 | 0.799564 | 0.79542 | 0.636336 | 0.011872073 |
| 75 | 0.791679 | 0.747345 | 0.676457 | 0.541165 | 0.018549786 |
| 76 | 0.760726 | 0.547723 | 0.748355 | 0.598684 | 0.051917626 |
| 77 | 0.950907 | 0.897656 | 0.767927 | 0.614341 | 0.004836578 |
| 78 | 0.763274 | 0.635044 | 0.761233 | 0.608987 | 0.032123312 |
| 79 | 0.769759 | 0.578859 | 0.765074 | 0.612059 | 0.043206863 |
| 80 | 0.742392 | 0.534522 | 0.730882 | 0.584705 | 0.056585277 |
| 81 | 0.776412 | 0.658398 | 0.771605 | 0.617284 | 0.027768282 |
| 82 | 0.74476 | 0.512395 | 0.722483 | 0.577986 | 0.063811258 |
| 83 | 2.329237 | 1.900658 | 0.813405 | 0.650724 | 5.92142E-09 |
| 84 | 0.882341 | 0.465876 | 0.716053 | 0.572843 | 0.081161259 |
| 85 | 0.909241 | 0.872872 | 0.833333 | 0.666667 | 0.006692555 |